

OTHER COLLEGE

FYJC - MATHEMATICS & STATISTICS

PAPER - I

LIMITS

Pg 01

LIMITS OF ALGEBRAIC FUNCTIONS

Pg 22

LIMITS OF TRIGONOMETRIC FUNCTIONS

Pg 39

LIMITS OF LOGARITHMIC

& EXPONENTIAL FUNCTIONS

LIMITS OF ALGEBRIAC FUNCTIONS

Q SET - 1

01. $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x(x - 5) + 6}$

ans : -3

02. $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{2x^2 - 3x - 9}$

ans : 1/9

03. $\lim_{x \rightarrow 1} \frac{x^2 - 8x + 7}{7x^2 - 6x - 1}$

ans : -3/4

04. $\lim_{x \rightarrow 1} \frac{3x(x^2 - 7x + 6)}{(x + 2)(x^2 - 4x + 3)}$

ans : 5/2

05. $\lim_{x \rightarrow -4} \frac{x^2 + 3x - 4}{x^2 + 9x + 20}$

ans : -5

06. $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x^2 - 5x + 6}$

ans : -32

07. $\lim_{x \rightarrow 3} \left(\frac{1}{x - 3} - \frac{3}{x^2 - 3x} \right)$

ans : 1/3

08. $\lim_{x \rightarrow 5} \left(\frac{1}{x - 5} - \frac{5}{x^2 - 5x} \right)$

ans : 1/5

09. $\lim_{x \rightarrow 2} \left(\frac{1}{x - 2} - \frac{4}{x^3 - 2x^2} \right)$

ans : 1

10. $\lim_{x \rightarrow 4} \left(\frac{1}{x^2 - 3x - 4} + \frac{1}{x^2 - 13x + 36} \right)$

11. $\lim_{x \rightarrow 3} \left(\frac{1}{x^2 - 11x + 24} + \frac{1}{x^2 - x - 6} \right)$

ans : -2/25

12. $\lim_{x \rightarrow 2} \left(\frac{1}{x - 2} - \frac{3}{x^2 - x - 2} \right)$

ans : 1/3

13. $\lim_{x \rightarrow 3} \left(\frac{1}{x - 3} - \frac{9x}{x^3 - 27} \right)$

ans : 0

14. $\lim_{x \rightarrow 2} \left(\frac{1}{x - 2} + \frac{6x}{8 - x^3} \right)$

ans : 0

Q SET - 2

01. $\lim_{x \rightarrow 1} \frac{\sqrt{x + 4} - \sqrt{5}}{x - 1}$

ans : 1/2√5

02. $\lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 1} - \sqrt{2}}{x - 1}$

ans : 1/√2

03. $\lim_{x \rightarrow 3} \frac{\sqrt{x^2 + 1} - \sqrt{10}}{x - 3}$

ans : 3/√10

04. $\lim_{x \rightarrow 3} \frac{x + 1 - \sqrt{x + 13}}{x - 3}$

ans : 7/8

05. $\lim_{x \rightarrow 4} \frac{x^2 + x - 20}{\sqrt{3x + 4} - 4}$

ans : 24

06. $\lim_{x \rightarrow 0} \frac{\sqrt{2 + x} - \sqrt{2}}{x\sqrt{2(2 + x)}}$

ans : 1/4√2

07. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x}$ ans : $3/\sqrt{10}$

04. $\lim_{x \rightarrow 1} \frac{3x^3 + 4x^2 - 6x - 1}{2x^3 - x - 1}$ ans : $11/5$

08. $\lim_{x \rightarrow 3} \frac{x^3 - 27}{\sqrt{x^2 + 7} - 4}$ ans : 36

05. $\lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 4}{x^3 - 2x^2 - 4x + 8}$ ans : $3/4$

09. $\lim_{x \rightarrow 4} \frac{2 - \sqrt{8-x}}{1 - \sqrt{5-x}}$ ans : $1/2$

06. $\lim_{x \rightarrow 1} \frac{x^3 - x^2 - x + 1}{x^3 + x^2 - 5x + 3}$ ans : $1/2$

10. $\lim_{x \rightarrow 7} \frac{4 - \sqrt{9+x}}{1 - \sqrt{8-x}}$ ans : $-1/4$

07. $\lim_{x \rightarrow 3} \frac{x^3 - 3x^2 - 9x + 27}{x^3 - 6x^2 - 9x}$ ans : 2

11. $\lim_{x \rightarrow 2} \frac{x^2 + \sqrt{x+2} - 6}{x^2 - 4}$ ans : $17/16$

Q SET - 4

12. $\lim_{x \rightarrow 3} \frac{x^2 + \sqrt{x+6} - 12}{x^2 - 9}$ ans : $37/36$

01. $\lim_{x \rightarrow a} \frac{x^{25} - a^{25}}{x^{15} - a^{15}}$ ans : $5a^{10}/3$

13. $\lim_{x \rightarrow 4} \frac{x^2 + \sqrt{x+5} - 19}{x^2 - 16}$ ans : $49/48$

02. $\lim_{x \rightarrow a} \frac{x^7 - a^7}{x^{11} - a^{11}}$ ans : $7/11a^4$

Q SET - 3

03. $\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16}$ ans : 6

01. $\lim_{x \rightarrow 2} \frac{x^3 + 3x^2 - 9x - 2}{x^3 - x - 6}$ ans : $15/11$

04. $\lim_{x \rightarrow 2} \frac{x^6 - 64}{x^{10} - 1024}$ ans : $3/80$

02. $\lim_{x \rightarrow 3} \frac{x^3 - x - 24}{x^3 + x^2 - 36}$ ans : $26/33$

05. $\lim_{x \rightarrow a} \frac{x^{-3} - a^{-3}}{x^{-7} - a^{-7}}$ ans : $3a^4/7$

03. $\lim_{x \rightarrow 3} \frac{x^3 - 4x - 15}{x^3 + x^2 - 6x - 18}$ ans : $23/27$

06. $\lim_{x \rightarrow 3} \frac{x^{-4} - 3^{-4}}{x^{-7} - 3^{-7}}$ ans : $108/7$

07. $\lim_{x \rightarrow 3} \frac{x^{1/4} - 3^{1/4}}{x^{1/3} - 3^{1/3}} \quad \text{ans: } 3^{11/12}/7$

08. $\lim_{x \rightarrow k} \frac{x^8 - k^8}{x - k} = 8$, find k

09. $\lim_{x \rightarrow k} \frac{x^5 - k^5}{x - k} = 80$, find k

10. $\lim_{x \rightarrow 1} \frac{x + x^2 + x^3 - 3}{x - 1} \quad \text{ans : } 6$

11. $\lim_{x \rightarrow 3} \frac{x + x^2 + x^3 - 39}{x - 3} \quad \text{ans : } 34$

SOLUTION TO Q SET - 1

01. $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x(x - 5) + 6}$

$$= \lim_{x \rightarrow 2} \frac{\begin{array}{c} -2 \quad +1 \\ \diagdown \quad / \\ x^2 - x - 2 \end{array}}{x^2 - 5x + 6}$$

$$= \lim_{x \rightarrow 2} \frac{x^2 - 2x + 1x - 2}{x^2 - 2x - 3x + 6}$$

$$= \lim_{x \rightarrow 2} \frac{x(x - 2) + 1(x - 2)}{x(x - 2) - 3(x - 2)}$$

$$= \lim_{x \rightarrow 2} \frac{\cancel{(x - 2)}(x + 1)}{\cancel{(x - 2)}(x - 3)}$$

$(x \rightarrow 2 ; x \neq 2 \therefore x - 2 \neq 0)$

$$= \lim_{x \rightarrow 2} \frac{x + 1}{x - 3}$$

$$= \frac{2 + 1}{2 - 3}$$

$$= -3$$

02. $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{2x^2 - 3x - 9}$

$$= \lim_{x \rightarrow 3} \frac{\begin{array}{c} -3 \quad -2 \\ \diagdown \quad / \\ x^2 - 5x + 6 \end{array}}{2x^2 - 3x - 9}$$

$$= \lim_{x \rightarrow 3} \frac{x^2 - 3x - 2x + 6}{2x^2 - 6x + 3x - 9}$$

$$= \lim_{x \rightarrow 3} \frac{x(x - 3) - 2(x - 3)}{2x(x - 3) + 3(x - 3)}$$

$$= \lim_{x \rightarrow 3} \frac{\cancel{(x - 3)}(x - 2)}{\cancel{(x - 3)}(2x + 3)}$$

$(x \rightarrow 3 ; x \neq 3 \therefore x - 3 \neq 0)$

CUT

$$= \lim_{x \rightarrow 3} \frac{x - 2}{2x + 3}$$

COPY

$$= \frac{3 - 2}{2(3) + 3}$$

PASTE

$$= \frac{1}{9}$$

03. $\lim_{x \rightarrow 1} \frac{x^2 - 8x + 7}{7x^2 - 6x - 1}$

$$= \lim_{x \rightarrow 1} \frac{\begin{array}{c} -1 \quad -7 \\ \diagdown \quad / \\ x^2 - 8x + 7 \end{array}}{7x^2 - 6x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{x^2 - 1x - 7x + 7}{7x^2 - 7x + 1x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{x(x - 1) - 7(x - 1)}{7x(x - 1) + 1(x - 1)}$$

$$= \lim_{x \rightarrow 1} \frac{\cancel{(x - 1)}(x - 7)}{\cancel{(x - 1)}(7x + 1)}$$

$(x \rightarrow 1 ; x \neq 1 \therefore x - 1 \neq 0)$

CUT

$$= \lim_{x \rightarrow 1} \frac{x - 7}{7x + 1}$$

COPY

$$= \frac{1 - 7}{7(1) + 1}$$

PASTE

$$= \frac{-6}{8}$$

$$= -3/4$$

$$04. \lim_{x \rightarrow 1} \frac{3x(x^2 - 7x + 6)}{(x+2)(x^2 - 4x + 3)}$$

$$= \lim_{x \rightarrow 1} \frac{3x \cdot \overset{-1}{\cancel{x}} \overset{-6}{\cancel{+6}}}{(x+2) \cdot (\overset{-3}{\cancel{x^2 - 4x}} + 3)}$$

$$= \lim_{x \rightarrow 1} \frac{3x \cdot (x^2 - 1x - 6x + 6)}{(x+2) \cdot (x^2 - 3x - 1x + 3)}$$

$$= \lim_{x \rightarrow 1} \frac{3x \cdot (x^2 - 1x - 6x + 6)}{(x+2) \cdot (x^2 - 3x - 1x + 3)}$$

$$= \lim_{x \rightarrow 1} \frac{3x \cdot [x(x-1) - 6(x-1)]}{(x+2) \cdot [x(x-3) - 1(x-3)]}$$

$$= \lim_{x \rightarrow 1} \frac{3x \cdot (x-1)(x-6)}{(x+2) \cdot (x-1)(x-3)} \quad \text{CUT}$$

$$= \lim_{x \rightarrow 1} \frac{3x \cdot (x-6)}{(x+2) \cdot (x-3)} \quad \text{COPY}$$

$$= \frac{3(1)(1-6)}{(1+2)(1-3)} \quad \text{PASTE}$$

$$= \frac{3(-5)}{3(-2)}$$

$$= \frac{5}{2}$$

$$05. \lim_{x \rightarrow -4} \frac{x^2 + 3x - 4}{x^2 + 9x + 20}$$

$$= \lim_{x \rightarrow -4} \frac{\overset{+4}{\cancel{x^2 + 3x}} \overset{-1}{\cancel{-4}}}{\overset{+4}{\cancel{x^2 + 9x}} \overset{+5}{\cancel{+20}}}$$

$$= \lim_{x \rightarrow -4} \frac{x^2 + 4x - 1x - 4}{x^2 + 4x + 5x + 20}$$

$$= \lim_{x \rightarrow -4} \frac{x(x+4) - 1(x+4)}{x(x+4) + 5(x+4)}$$

$$= \lim_{x \rightarrow -4} \frac{\cancel{(x+4)}(x-1)}{\cancel{(x+4)}(x+5)} \quad \text{CUT}$$

($x \rightarrow -4$; $x \neq -4 \therefore x+4 \neq 0$)

$$= \lim_{x \rightarrow -4} \frac{x-1}{x+5} \quad \text{COPY}$$

$$= \frac{-4-1}{-4+5} \quad \text{PASTE}$$

$$= -5$$

$$06. \lim_{x \rightarrow 2} \frac{x^4 - 16}{x^2 - 5x + 6}$$

$$= \lim_{x \rightarrow 2} \frac{(x^2 - 4)(x^2 + 4)}{\overset{-2}{\cancel{x^2 - 5x}} \overset{-3}{\cancel{+6}}}$$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x+2) \cdot (x^2 + 4)}{x^2 - 2x - 3x + 6}$$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x+2) \cdot (x^2 + 4)}{x(x-2) - 3(x-2)}$$

$$= \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(x+2) \cdot (x^2 + 4)}{\cancel{(x-2)}(x-3)} \quad \text{CUT}$$

($x \rightarrow 2$; $x \neq 2 \therefore x-2 \neq 0$)

$$= \lim_{x \rightarrow 2} \frac{(x+2) \cdot (x^2 + 4)}{x-3} \quad \text{COPY}$$

$$= \frac{2+2)(4+4)}{2-3} \quad \text{PASTE}$$

$$= \frac{4(8)}{-1} = -32$$

$$07. \lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{3}{x^2-3x} \right)$$

$$= \lim_{x \rightarrow 2} \frac{x+2}{x^2}$$

COPY

$$= \lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{3}{x(x-3)} \right)$$

$$= \frac{2+2}{2^2}$$

PASTE

$$= \lim_{x \rightarrow 3} \frac{\cancel{x-3}}{x(\cancel{x-3})}$$

CUT

$$= 1$$

$$(x \rightarrow 3; x \neq 3 \therefore x-3 \neq 0)$$

$$= \lim_{x \rightarrow 3} \frac{1}{x}$$

COPY

10.

$$\lim_{x \rightarrow 4} \left(\frac{1}{x^2-3x-4} + \frac{1}{x^2-13x+36} \right)$$

$$= \frac{1}{3}$$

PASTE

$$= \lim_{x \rightarrow 4} \left(\frac{1}{\underbrace{x^2-3x-4}_{-4 \quad +1}} + \frac{1}{\underbrace{x^2-13x+36}_{-9 \quad -4}} \right)$$

$$08. \lim_{x \rightarrow 5} \left(\frac{1}{x-5} - \frac{5}{x^2-5x} \right)$$

$$= \lim_{x \rightarrow 4} \left(\frac{1}{x^2-4x+x-4} + \frac{1}{x^2-9x-4x+36} \right)$$

$$= \lim_{x \rightarrow 5} \left(\frac{1}{x-5} - \frac{5}{x(x-5)} \right)$$

$$= \lim_{x \rightarrow 4} \left(\frac{1}{x(x-4)+1(x-4)} + \frac{1}{x(x-9)-4(x-9)} \right)$$

$$= \lim_{x \rightarrow 5} \frac{\cancel{x-5}}{x(\cancel{x-5})}$$

CUT

$$= \lim_{x \rightarrow 4} \left(\frac{1}{(x-4)(x+1)} + \frac{1}{(x-9)(x-4)} \right)$$

$$= \lim_{x \rightarrow 5} \frac{1}{x}$$

COPY

$$= \lim_{x \rightarrow 4} \frac{x-9+x+1}{(x-4)(x+1)(x-9)}$$

$$= \frac{1}{5}$$

PASTE

$$= \lim_{x \rightarrow 4} \frac{2x-8}{(x-4)(x+1)(x-9)}$$

$$09. \lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{4}{x^3-2x^2} \right)$$

$$= \lim_{x \rightarrow 4} \frac{\cancel{2(x-4)}}{(x-\cancel{4})(x+1)(x-9)}$$

CUT

$$= \lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{4}{x^2(x-2)} \right)$$

$$(x \rightarrow 4; x \neq 4 \therefore x-4 \neq 0)$$

$$= \lim_{x \rightarrow 4} \frac{2}{(x+1)(x-9)}$$

COPY

$$= \lim_{x \rightarrow 2} \frac{x^2-4}{x^2(x-2)}$$

$$= \frac{2}{(4+1)(4-9)}$$

PASTE

$$= \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(x+2)}{x^2(\cancel{x-2})}$$

CUT

$$= \frac{2}{(5)(-5)}$$

$$= -2/25$$

11.

$$\begin{aligned} \lim_{x \rightarrow 3} \left(\frac{1}{x^2 - 11x + 24} + \frac{1}{x^2 - x - 6} \right) \\ = \lim_{x \rightarrow 3} \left(\frac{1}{\begin{array}{c} x^2 - 11x + 24 \\ \underbrace{\quad\quad\quad}_{-3 \quad -8} \end{array}} + \frac{1}{\begin{array}{c} x^2 - x - 6 \\ \underbrace{\quad\quad\quad}_{-3 \quad +2} \end{array}} \right) \\ = \lim_{x \rightarrow 3} \left(\frac{1}{x^2 - 3x - 8x + 24} + \frac{1}{x^2 - 3x + 2x - 6} \right) \\ = \lim_{x \rightarrow 3} \left(\frac{1}{x(x-3) - 8(x-3)} + \frac{1}{x(x-3) + 2(x-3)} \right) \end{aligned}$$

$$= \lim_{x \rightarrow 3} \left(\frac{1}{(x-3)(x-8)} + \frac{1}{(x-3)(x+2)} \right)$$

$$= \lim_{x \rightarrow 3} \frac{x+2 + x-8}{(x-3)(x-8)(x+2)}$$

$$= \lim_{x \rightarrow 3} \frac{2x-6}{(x-3)(x-8)(x+2)}$$

$$= \lim_{x \rightarrow 3} \frac{\cancel{2(x-3)}}{(x-\cancel{3})(x-8)(x+2)} \quad \text{CUT}$$

(x → 3 ; x ≠ 3 ∴ x - 3 ≠ 0)

$$= \lim_{x \rightarrow 3} \frac{2}{(x-8)(x+2)} \quad \text{COPY}$$

$$= \frac{2}{(3-8)(3+2)} \quad \text{PASTE}$$

$$= \frac{2}{(-5)(5)} = -2 / 25$$

12.

$$\lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{3}{x^2 - x - 2} \right)$$

$$= \lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{3}{x^2 - 2x + 1x - 2} \right)$$

$$= \lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{3}{x(x-2) + 1(x-2)} \right)$$

$$= \lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{3}{(x-2)(x+1)} \right)$$

$$= \lim_{x \rightarrow 2} \frac{x+1-3}{(x-2)(x+1)}$$

$$= \lim_{x \rightarrow 2} \frac{\cancel{x-2}}{(\cancel{x-2})(x+1)} \quad \text{CUT}$$

(x → 2 ; x ≠ 2 ∴ x - 2 ≠ 0)

$$= \lim_{x \rightarrow 2} \frac{1}{x+1} \quad \text{COPY}$$

$$= \frac{1}{2+1} \quad \text{PASTE}$$

$$= 1 / 3$$

13.

$$\lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{9x}{x^3 - 27} \right)$$

$$= \lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{9x}{x^3 - 3^3} \right)$$

$$= \lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{9x}{(x-3)(x^2 + 3x + 9)} \right)$$

$$= \lim_{x \rightarrow 3} \frac{x^2 + 3x + 9 - 9x}{(x-3)(x^2 + 3x + 9)}$$

$$= \lim_{x \rightarrow 3} \frac{\begin{array}{c} -3 \quad -3 \\ \underbrace{\quad\quad\quad} \end{array} x^2 - 6x + 9}{(x-3)(x^2 + 3x + 9)}$$

$$= \lim_{x \rightarrow 3} \frac{x^2 - 3x - 3x + 9}{(x-3)(x^2 + 3x + 9)}$$

$$= \lim_{x \rightarrow 3} \frac{x(x-3) - 3(x-3)}{(x-3)(x^2 + 3x + 9)}$$

$$= \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x-3)}{\cancel{(x-3)}(x^2 + 3x + 9)}$$

$$(x \rightarrow 3 ; x \neq 3 \therefore x - 3 \neq 0)$$

$$= \lim_{x \rightarrow 3} \frac{x-3}{x^2 + 3x + 9}$$

$$= \frac{3-3}{3^2 + 3(3) + 9}$$

$$= 0$$

14.

$$\lim_{x \rightarrow 2} \left(\frac{1}{x-2} + \frac{6x}{8-x^3} \right)$$

$$= \lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{6x}{x^3 - 2^3} \right)$$

$$= \lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{6x}{(x-2)(x^2 + 2x + 4)} \right)$$

$$= \lim_{x \rightarrow 2} \frac{x^2 + 2x + 4 - 6x}{(x-2)(x^2 + 2x + 4)}$$

$$= \lim_{x \rightarrow 2} \frac{\begin{array}{c} -2 \quad -2 \\ \diagdown \quad / \\ x^2 - 4x + 4 \end{array}}{(x-2)(x^2 + 2x + 4)}$$

$$= \lim_{x \rightarrow 2} \frac{x^2 - 2x - 2x + 4}{(x-2)(x^2 + 2x + 4)}$$

$$= \lim_{x \rightarrow 2} \frac{x(x-2) - 2(x-2)}{(x-2)(x^2 + 2x + 4)}$$

$$= \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(x-2)}{\cancel{(x-2)}(x^2 + 2x + 4)}$$

$$(x \rightarrow 2 ; x \neq 2 \therefore x - 2 \neq 0)$$

$$= \lim_{x \rightarrow 2} \frac{x-2}{x^2 + 2x + 4} \quad \text{COPY}$$

$$= \frac{2-2}{2^2 + 2(2) + 4} \quad \text{PASTE}$$

$$= 0$$

CUT

COPY

PASTE

SOLUTION TO Q SET - 2

01.

$$\lim_{x \rightarrow 1} \frac{\sqrt{x+4} - \sqrt{5}}{x-1}$$

$$= \lim_{x \rightarrow 1} \frac{\sqrt{x+4} - \sqrt{5}}{x-1} \cdot \frac{\sqrt{x+4} + \sqrt{5}}{\sqrt{x+4} + \sqrt{5}}$$

$$= \lim_{x \rightarrow 1} \frac{x+4-5}{x-1} \cdot \frac{1}{\sqrt{x+4} + \sqrt{5}}$$

$$= \lim_{x \rightarrow 1} \frac{\cancel{x-1}}{\cancel{x-1}} \cdot \frac{1}{\sqrt{x+4} + \sqrt{5}} \quad \text{CUT}$$

$$(x \rightarrow 1 ; x \neq 1 \therefore x - 1 \neq 0)$$

$$= \lim_{x \rightarrow 1} \frac{1}{\sqrt{x+4} + \sqrt{5}} \quad \text{COPY}$$

$$= \frac{1}{\sqrt{1+4} + \sqrt{5}} \quad \text{PASTE}$$

$$= \frac{1}{\sqrt{5} + \sqrt{5}}$$

$$= \frac{1}{2\sqrt{5}}$$

CUT

02.

$$\begin{aligned} & \lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 1} - \sqrt{2}}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 1} - \sqrt{2}}{x - 1} \cdot \frac{\sqrt{x^2 + 1} + \sqrt{2}}{\sqrt{x^2 + 1} + \sqrt{2}} \\ &= \lim_{x \rightarrow 1} \frac{x^2 + 1 - 2}{x - 1} \cdot \frac{1}{\sqrt{x^2 + 1} + \sqrt{2}} \\ &= \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} \cdot \frac{1}{\sqrt{x^2 + 1} + \sqrt{2}} \\ &= \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(x+1)}{x-1} \cdot \frac{1}{\sqrt{x^2 + 1} + \sqrt{2}} \quad \text{CUT} \\ & \quad (x \rightarrow 1 ; x \neq 1 \therefore x - 1 \neq 0) \end{aligned}$$

$$\begin{aligned} &= \lim_{x \rightarrow 1} \frac{1}{\sqrt{x^2 + 1} + \sqrt{2}} \quad \text{COPY} \\ &= \frac{1}{\sqrt{1 + 4} + \sqrt{5}} \quad \text{PASTE} \\ &= \frac{1}{\sqrt{5} + \sqrt{5}} \\ &= \frac{1}{2\sqrt{5}} \end{aligned}$$

03.

$$\begin{aligned} & \lim_{x \rightarrow 3} \frac{\sqrt{x^2 + 1} - \sqrt{10}}{x - 3} \\ &= \lim_{x \rightarrow 3} \frac{\sqrt{x^2 + 1} - \sqrt{10}}{x - 3} \cdot \frac{\sqrt{x^2 + 1} + \sqrt{10}}{\sqrt{x^2 + 1} + \sqrt{10}} \\ &= \lim_{x \rightarrow 3} \frac{x^2 + 1 - 10}{x - 3} \cdot \frac{1}{\sqrt{x^2 + 1} + \sqrt{10}} \\ &= \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} \cdot \frac{1}{\sqrt{x^2 + 1} + \sqrt{10}} \end{aligned}$$

$$\begin{aligned} &= \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x+3)}{x-3} \cdot \frac{1}{\sqrt{x^2 + 1} + \sqrt{10}} \quad \text{CUT} \\ & \quad (x \rightarrow 3 ; x \neq 3 \therefore x - 3 \neq 0) \end{aligned}$$

$$\begin{aligned} &= \lim_{x \rightarrow 3} \frac{1}{\sqrt{x^2 + 1} + \sqrt{10}} \quad \text{COPY} \\ &= \frac{1}{\sqrt{9 + 1} + \sqrt{10}} \quad \text{PASTE} \\ &= \frac{1}{\sqrt{10} + \sqrt{10}} \\ &= \frac{1}{2\sqrt{10}} \end{aligned}$$

04.

$$\begin{aligned} & \lim_{x \rightarrow 3} \frac{x + 1 - \sqrt{x + 13}}{x - 3} \\ &= \lim_{x \rightarrow 3} \frac{x + 1 - \sqrt{x + 13}}{x - 3} \cdot \frac{x + 1 + \sqrt{x + 13}}{x + 1 + \sqrt{x + 13}} \\ &= \lim_{x \rightarrow 3} \frac{(x + 1)^2 - (x + 13)}{x - 3} \cdot \frac{1}{x + 1 + \sqrt{x + 13}} \\ &= \lim_{x \rightarrow 3} \frac{x^2 + 2x + 1 - x - 13}{x - 3} \cdot \frac{1}{x + 1 + \sqrt{x + 13}} \end{aligned}$$

$$\begin{aligned} &= \lim_{x \rightarrow 3} \frac{\overset{+4}{x^2 + x} - \overset{-3}{12}}{x - 3} \cdot \frac{1}{x + 1 + \sqrt{x + 13}} \\ &= \lim_{x \rightarrow 3} \frac{x^2 + 4x - 3x - 12}{x - 3} \cdot \frac{1}{x + 1 + \sqrt{x + 13}} \\ &= \lim_{x \rightarrow 3} \frac{x(x + 4) - 3(x + 4)}{x - 3} \cdot \frac{1}{x + 1 + \sqrt{x + 13}} \\ &= \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x+4)}{x-3} \cdot \frac{1}{x + 1 + \sqrt{x + 13}} \quad \text{CUT} \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 3} \frac{x+4}{x+1+\sqrt{x+13}} && \text{COPY} \\
 &= \frac{3+4}{3+1+\sqrt{3+13}} && \text{PASTE} \\
 &= \frac{7}{4+4} \\
 &= 7/8
 \end{aligned}$$

05.

$$\begin{aligned}
 &\lim_{x \rightarrow 4} \frac{x^2+x-20}{\sqrt{3x+4}-4} \\
 &= \lim_{x \rightarrow 4} \frac{x^2+5x-4x-20}{\sqrt{3x+4}-4} \cdot \frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4} \\
 &= \lim_{x \rightarrow 4} \frac{x(x+5)-4(x+5)}{3x+4-16} \cdot \frac{\sqrt{3x+4}+4}{1} \\
 &= \lim_{x \rightarrow 4} \frac{(x+5)(x-4)}{3x-12} \cdot \frac{\sqrt{3x+4}+4}{1} \\
 &= \lim_{x \rightarrow 4} \frac{(x+5)(\cancel{x-4})}{3(\cancel{x-4})} \cdot \frac{\sqrt{3x+4}+4}{1} && \text{CUT} \\
 & \quad (x \rightarrow 4 ; x \neq 4 \therefore x-4 \neq 0)
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 4} \frac{x+5}{3} \cdot \frac{\sqrt{3x+4}+4}{1} && \text{COPY} \\
 &= \frac{4+5}{3} \cdot \frac{\sqrt{3(4)+4}+4}{1} && \text{PASTE} \\
 &= \frac{9}{3} \cdot \frac{\sqrt{12+4}+4}{1} \\
 &= 3(4+4) \\
 &= 24
 \end{aligned}$$

06.

$$\begin{aligned}
 &\lim_{x \rightarrow 0} \frac{\sqrt{2+x}-\sqrt{2}}{x\sqrt{2(2+x)}} \\
 &= \lim_{x \rightarrow 0} \frac{\sqrt{2+x}-\sqrt{2}}{x\sqrt{2(2+x)}} \cdot \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}} \\
 &= \lim_{x \rightarrow 0} \frac{2+x-2}{x\sqrt{2(2+x)}} \cdot \frac{1}{\sqrt{2+x}+\sqrt{2}} \\
 &= \lim_{x \rightarrow 0} \frac{\cancel{-x}}{\cancel{-x}\sqrt{2(2+x)}} \cdot \frac{1}{\sqrt{2+x}+\sqrt{2}} && \text{CUT} \\
 & \quad (x \rightarrow 0 ; x \neq 0)
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{1}{\sqrt{2(2+x)}} \cdot \frac{1}{\sqrt{2+x}+\sqrt{2}} && \text{COPY} \\
 &= \frac{1}{\sqrt{2(2+0)}} \cdot \frac{1}{\sqrt{2+0}+\sqrt{2}} && \text{PASTE} \\
 &= \frac{1}{2} \cdot \frac{1}{\sqrt{2}+\sqrt{2}} \\
 &= \frac{1}{2} \cdot \frac{1}{2\sqrt{2}} \\
 &= \frac{1}{4\sqrt{2}}
 \end{aligned}$$

07.

$$\begin{aligned}
 &\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2}-1}{x} \\
 &= \lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2}-1}{x} \cdot \frac{\sqrt{1+x+x^2}+1}{\sqrt{1+x+x^2}+1} \\
 &= \lim_{x \rightarrow 0} \frac{1+x+x^2-1}{x} \cdot \frac{1}{\sqrt{1+x+x^2}+1} \\
 &= \lim_{x \rightarrow 0} \frac{x+x^2}{x} \cdot \frac{1}{\sqrt{1+x+x^2}+1}
 \end{aligned}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{\cancel{x}(x+1)}{\cancel{x}} \frac{1}{\sqrt{1+x+x^2} + 1} \quad \text{CUT} &= \frac{27}{6} \cdot \frac{4+4}{1} \\
&(x \rightarrow 0; x \neq 0) &= \frac{9}{2} (8) = 36 \\
&= \lim_{x \rightarrow 0} \frac{x+1}{\sqrt{1+x+x^2} + 1} \quad \text{COPY} \\
&= \frac{0+1}{\sqrt{1+0+0} + 1} \quad \text{PASTE} \\
&= \frac{1}{1+1} = \frac{1}{2}
\end{aligned}$$

08.

$$\begin{aligned}
&\lim_{x \rightarrow 3} \frac{x^3 - 27}{\sqrt{x^2 + 7} - 4} \\
&= \lim_{x \rightarrow 3} \frac{x^3 - 3^3}{\sqrt{x^2 + 7} - 4} \frac{\sqrt{x^2 + 7} + 4}{\sqrt{x^2 + 7} + 4} \\
&= \lim_{x \rightarrow 3} \frac{(x-3)(x^2 + 3x + 9)}{x^2 + 7 - 16} \frac{\sqrt{x^2 + 7} + 4}{1} \\
&= \lim_{x \rightarrow 3} \frac{(x-3)(x^2 + 3x + 9)}{x^2 - 9} \frac{\sqrt{x^2 + 7} + 4}{1} \\
&= \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x^2 + 3x + 9)}{\cancel{(x-3)}(x+3)} \frac{\sqrt{x^2 + 7} + 4}{1} \\
&(x \rightarrow 3; x \neq 3 \therefore x - 3 \neq 0) \\
&= \lim_{x \rightarrow 3} \frac{x^2 + 3x + 9}{x + 3} \frac{\sqrt{x^2 + 7} + 4}{1} \\
&= \frac{3^2 + 3(3) + 9}{3 + 3} \frac{\sqrt{3^2 + 7} + 4}{1} \\
&= \frac{9 + 9 + 9}{6} \frac{\sqrt{9 + 7} + 4}{1}
\end{aligned}$$

$$\begin{aligned}
09. \quad & \lim_{x \rightarrow 4} \frac{2 - \sqrt{8-x}}{1 - \sqrt{5-x}} \\
&= \lim_{x \rightarrow 4} \frac{2 - \sqrt{8-x}}{1 - \sqrt{5-x}} \cdot \frac{2 + \sqrt{8-x}}{2 + \sqrt{8-x}} \cdot \frac{1 + \sqrt{5-x}}{1 + \sqrt{5-x}} \\
&= \lim_{x \rightarrow 4} \frac{4 - (8-x)}{1 - (5-x)} \cdot \frac{1}{2 + \sqrt{8-x}} \cdot \frac{1 + \sqrt{5-x}}{1} \\
&= \lim_{x \rightarrow 4} \frac{4 - (8-x)}{1 - (5-x)} \cdot \frac{1 + \sqrt{5-x}}{2 + \sqrt{8-x}} \\
&= \lim_{x \rightarrow 4} \frac{4 - 8 + x}{1 - 5 + x} \cdot \frac{1 + \sqrt{5-x}}{2 + \sqrt{8-x}} \\
&= \lim_{x \rightarrow 4} \frac{x-4}{x-4} \cdot \frac{1 + \sqrt{5-x}}{2 + \sqrt{8-x}} \quad \text{CUT} \\
&\quad (x \rightarrow 4 ; x \neq 4 \therefore x-4 \neq 0) \\
&= \lim_{x \rightarrow 4} \frac{1 + \sqrt{5-x}}{2 + \sqrt{8-x}} \quad \text{COPY} \\
&= \frac{1 + \sqrt{5-4}}{2 + \sqrt{8-4}} \quad \text{PASTE} \\
&= \frac{1 + \sqrt{1}}{2 + \sqrt{4}} \\
&= \frac{1+1}{2+2} \\
&= \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
10. \quad & \lim_{x \rightarrow 7} \frac{4 - \sqrt{9+x}}{1 - \sqrt{8-x}} \\
&= \lim_{x \rightarrow 7} \frac{4 - \sqrt{9+x}}{1 - \sqrt{8-x}} \cdot \frac{4 + \sqrt{9+x}}{4 + \sqrt{9+x}} \cdot \frac{1 + \sqrt{8-x}}{1 + \sqrt{8-x}} \\
&= \lim_{x \rightarrow 7} \frac{16 - (9+x)}{1 - (8-x)} \cdot \frac{1}{4 + \sqrt{9+x}} \cdot \frac{1 + \sqrt{8-x}}{1} \\
&= \lim_{x \rightarrow 7} \frac{16 - 9 - x}{1 - 8 + x} \cdot \frac{1 + \sqrt{8-x}}{4 + \sqrt{9+x}} \\
&= \lim_{x \rightarrow 7} \frac{7-x}{-7+x} \cdot \frac{1 + \sqrt{8-x}}{4 + \sqrt{9+x}}
\end{aligned}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 7} \frac{\cancel{x-7}}{\cancel{x-7}} \cdot \frac{1 + \sqrt{8-x}}{4 + \sqrt{9+x}} && \text{CUT} && = \frac{-1 + \sqrt{1}}{4 + \sqrt{16}} \\
& && (x \rightarrow 7 ; x \neq 7 \therefore x-7 \neq 0) && \\
&= \lim_{x \rightarrow 7} \frac{-1 + \sqrt{8-x}}{4 + \sqrt{9+x}} && \text{COPY} && = \frac{-1 + 1}{4 + 4} \\
&= \frac{-1 + \sqrt{8-7}}{4 + \sqrt{9+7}} && \text{PASTE} && = \frac{-1}{4}
\end{aligned}$$

00. $\lim_{x \rightarrow 1} \frac{\sqrt{x+8} - \sqrt{8x+1}}{\sqrt{5-x} - \sqrt{7x-3}}$ (EXTRA SUM)

$$= \lim_{x \rightarrow 1} \frac{\sqrt{x+8} - \sqrt{8x+1}}{\sqrt{5-x} - \sqrt{7x-3}} \cdot \frac{\sqrt{x+8} + \sqrt{8x+1}}{\sqrt{x+8} + \sqrt{8x+1}} \cdot \frac{\sqrt{5-x} + \sqrt{7x-3}}{\sqrt{5-x} + \sqrt{7x-3}}$$

$$= \lim_{x \rightarrow 1} \frac{x+8 - (8x+1)}{5-x - (7x-3)} \cdot \frac{\sqrt{5-x} + \sqrt{7x-3}}{\sqrt{x+8} + \sqrt{8x+1}}$$

$$= \lim_{x \rightarrow 1} \frac{x+8 - 8x-1}{5-x - 7x+3} \cdot \frac{\sqrt{5-x} + \sqrt{7x-3}}{\sqrt{x+8} + \sqrt{8x+1}}$$

$$= \lim_{x \rightarrow 1} \frac{-7x+7}{-8x+8} \cdot \frac{\sqrt{5-x} + \sqrt{7x-3}}{\sqrt{x+8} + \sqrt{8x+1}}$$

$$= \lim_{x \rightarrow 1} \frac{\cancel{-7(x-1)}}{\cancel{-8(x-1)}} \cdot \frac{\sqrt{5-x} + \sqrt{7x-3}}{\sqrt{x+8} + \sqrt{8x+1}} \quad \text{CUT}$$

$$(x \rightarrow 1 ; x \neq 1 \therefore x-1 \neq 0)$$

$$= \lim_{x \rightarrow 1} \frac{7}{8} \cdot \frac{\sqrt{5-x} + \sqrt{7x-3}}{\sqrt{x+8} + \sqrt{8x+1}} \quad \text{COPY}$$

$$= \frac{7}{8} \cdot \frac{\sqrt{5-1} + \sqrt{7-3}}{\sqrt{1+8} + \sqrt{8+1}} \quad \text{PASTE}$$

$$= \frac{7}{8} \cdot \frac{2 + 2}{3 + 3}$$

$$= \frac{7}{8} \times \frac{4}{6}$$

$$= 7 / 12$$

$$11. \lim_{x \rightarrow 2} \frac{x^2 + \sqrt{x+2} - 6}{x^2 - 4}$$

$$= \lim_{x \rightarrow 2} \frac{x^2 - 4 + \sqrt{x+2} - 2}{x^2 - 4}$$

$$= \lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - 4} + \frac{\sqrt{x+2} - 2}{x^2 - 4}$$

$$= \lim_{x \rightarrow 2} 1 + \frac{\sqrt{x+2} - 2}{x^2 - 4} \cdot \frac{\sqrt{x+2} + 2}{\sqrt{x+2} + 2}$$

$$= \lim_{x \rightarrow 2} 1 + \frac{x+2-4}{x^2-4} \cdot \frac{1}{\sqrt{x+2}+2}$$

$$= \lim_{x \rightarrow 2} 1 + \frac{\cancel{x}-2}{(\cancel{x}-2)(x+2)} \cdot \frac{1}{\sqrt{x+2}+2} \quad \text{CUT}$$

$(x \rightarrow 2 ; x \neq 2 \therefore x-2 \neq 0)$

$$= \lim_{x \rightarrow 2} 1 + \frac{1}{x+2} \cdot \frac{1}{\sqrt{x+2}+2}$$

COPY

PASTE

$$= 1 + \frac{1}{2+2} \cdot \frac{1}{\sqrt{2+2}+2}$$

$$= 1 + \frac{1}{4} \times \frac{1}{2+2}$$

$$= 1 + \frac{1}{16}$$



$$= 17/16$$

$$12. \lim_{x \rightarrow 3} \frac{x^2 + \sqrt{x+6} - 12}{x^2 - 9}$$

$$= \lim_{x \rightarrow 3} \frac{x^2 - 9 + \sqrt{x+6} - 3}{x^2 - 9}$$

$$= \lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 9} + \frac{\sqrt{x+6} - 3}{x^2 - 9}$$

$$= \lim_{x \rightarrow 3} 1 + \frac{\sqrt{x+6} - 3}{x^2 - 9} \cdot \frac{\sqrt{x+6} + 3}{\sqrt{x+6} + 3}$$

$$= \lim_{x \rightarrow 3} 1 + \frac{x+6-9}{x^2-9} \cdot \frac{1}{\sqrt{x+6}+3}$$

$$= \lim_{x \rightarrow 3} 1 + \frac{\cancel{x}+3}{(\cancel{x}+3)(x+3)} \cdot \frac{1}{\sqrt{x+6}+3} \quad \text{CUT}$$

$(x \rightarrow 3 ; x \neq 3 \therefore x+3 \neq 0)$

COPY

PASTE

$$= \lim_{x \rightarrow 3} 1 + \frac{1}{x+3} \cdot \frac{1}{\sqrt{x+6}+3}$$

$$= 1 + \frac{1}{3+3} \cdot \frac{1}{\sqrt{3+6}+3}$$

$$= 1 + \frac{1}{6} \cdot \frac{1}{3+3}$$

$$= 1 + \frac{1}{36}$$

$$= 37/36$$



$$13. \lim_{x \rightarrow 4} \frac{x^2 + \sqrt{x+5} - 19}{x^2 - 16}$$

$$= \lim_{x \rightarrow 4} \frac{x^2 - 16 + \sqrt{x+5} - 3}{x^2 - 16}$$

$$= \lim_{x \rightarrow 4} \frac{x^2 - 16}{x^2 - 16} + \frac{\sqrt{x+5} - 3}{x^2 - 16}$$

$$= \lim_{x \rightarrow 4} \left(1 + \frac{\sqrt{x+5} - 3}{x^2 - 16} \cdot \frac{\sqrt{x+5} + 3}{\sqrt{x+5} + 3} \right)$$

$$= \lim_{x \rightarrow 4} \left(1 + \frac{x+5-9}{x^2-16} \cdot \frac{1}{\sqrt{x+5}+3} \right)$$

$$= \lim_{x \rightarrow 4} \left(1 + \frac{\cancel{x-4}}{(x-4)(x+4)} \cdot \frac{1}{\sqrt{x+5}+3} \right) \text{ CUT}$$

$(x \rightarrow 4; x \neq 4 \therefore x-4 \neq 0)$

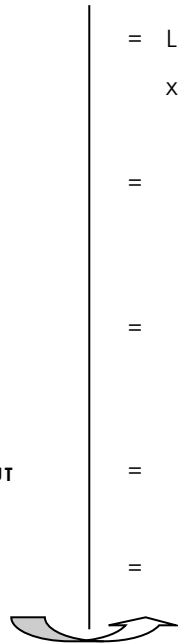
$$= \lim_{x \rightarrow 4} \left(1 + \frac{1}{x+4} \cdot \frac{1}{\sqrt{x+5}+3} \right) \text{ COPY}$$

$$= 1 + \frac{1}{4+4} \cdot \frac{1}{\sqrt{4+5}+3} \text{ PASTE}$$

$$= 1 + \frac{1}{8} \cdot \frac{1}{3+3}$$

$$= 1 + \frac{1}{48}$$

$$= \frac{49}{48}$$



SOLUTION TO Q SET - 3

$$01. \lim_{x \rightarrow 2} \frac{x^3 + 3x^2 - 9x - 2}{x^3 - x - 6}$$

$$x^3 + 3x^2 - 9x - 2 = (x-2)(?)$$

2	1	3	-9	-2
		2	10	2
	1	5	1	0

$$x^3 + 3x^2 - 9x - 2 = (x-2)(x^2 + 5x + 1)$$

$$x^3 - x - 6 = (x-2)(?)$$

2	1	0	-1	-6
		2	4	6
	1	2	3	0

$$x^3 - x - 6 = (x-2)(x^2 + 2x + 3)$$

BACK INTO THE SUM

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 5x + 1)}{(x-2)(x^2 + 2x + 3)} \text{ CUT}$$

$$(x \rightarrow 2; x \neq 2 \therefore x-2 \neq 0)$$

$$= \lim_{x \rightarrow 2} \frac{x^2 + 5x + 1}{x^2 + 2x + 3} \text{ COPY}$$

$$= \frac{2^2 + 5(2) + 1}{2^2 + 2(2) + 3} \text{ PASTE}$$

$$= \frac{4 + 10 + 1}{4 + 4 + 3}$$

$$02. \quad \lim_{x \rightarrow 3} \frac{x^3 - x - 24}{x^3 + x^2 - 36}$$

$$x^3 - x - 24 = (x - 3) (?)$$

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -1 & -24 & \\ & & 3 & 9 & 24 & \\ \hline & 1 & 3 & 8 & 0 & \end{array}$$

$$x^3 - x - 24 = (x - 3) (x^2 + 3x + 8)$$

$$x^3 + x^2 - 36 = (x - 2) (?)$$

$$\begin{array}{r|rrrrr} 3 & 1 & 1 & 0 & -36 & \\ & & 3 & 12 & 36 & \\ \hline & 1 & 4 & 12 & 0 & \end{array}$$

$$x^3 + x^2 - 36 = (x - 3) (x^2 + 4x + 12)$$

$$03. \quad \lim_{x \rightarrow 3} \frac{x^3 - 4x - 15}{x^3 + x^2 - 6x - 18}$$

$$x^3 - 4x - 15 = (x - 3) (?)$$

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -4 & -15 & \\ & & 3 & 9 & 15 & \\ \hline & 1 & 3 & 5 & 0 & \end{array}$$

$$x^3 - 4x - 15 = (x - 3) (x^2 + 3x + 5)$$

$$x^3 + x^2 - 6x - 18 = (x - 2) (?)$$

$$\begin{array}{r|rrrrr} 3 & 1 & 1 & -6 & -18 & \\ & & 3 & 12 & 18 & \\ \hline & 1 & 4 & 6 & 0 & \end{array}$$

$$\begin{aligned} x^3 + x^2 - 6x - 18 \\ = (x - 2) (x^2 + 4x + 6) \end{aligned}$$

BACK INTO THE SUM

$$= \lim_{x \rightarrow 3} \frac{\cancel{(x-3)} (x^2 + 3x + 8)}{\cancel{(x-3)} (x^2 + 4x + 12)}$$

CUT

$$(x \rightarrow 3 ; x \neq 3 \therefore x - 3 \neq 0)$$

$$= \lim_{x \rightarrow 3} \frac{x^2 + 3x + 8}{x^2 + 4x + 12}$$

COPY

$$= \frac{3^2 + 3(3) + 8}{3^2 + 4(3) + 12}$$

PASTE

$$= \frac{9 + 9 + 8}{9 + 12 + 12}$$

$$= \frac{26}{33}$$

BACK INTO THE SUM

$$= \lim_{x \rightarrow 3} \frac{\cancel{(x-3)} (x^2 + 3x + 5)}{\cancel{(x-3)} (x^2 + 4x + 6)}$$

CUT

$$(x \rightarrow 3 ; x \neq 3 \therefore x - 3 \neq 0)$$

$$= \lim_{x \rightarrow 3} \frac{x^2 + 3x + 5}{x^2 + 4x + 6}$$

COPY

$$= \frac{3^2 + 3(3) + 5}{3^2 + 4(3) + 6}$$

PASTE

$$= \frac{9 + 9 + 5}{9 + 12 + 6}$$

$$= \frac{23}{27}$$

$$04. \lim_{x \rightarrow 1} \frac{3x^3 + 4x^2 - 6x - 1}{2x^3 - x - 1}$$

$$3x^3 + 4x^2 - 6x - 1 = (x - 1) (?)$$

$$\begin{array}{r|rrrr} 1 & 3 & 4 & -6 & -1 \\ & & 3 & 7 & 1 \\ \hline & 3 & 7 & 1 & 0 \end{array}$$

$$\begin{aligned} 3x^3 + 4x^2 - 6x - 1 &= (x - 1) (3x^2 + 7x + 1) \end{aligned}$$

$$2x^3 - x - 1 = (x - 1) (?)$$

$$\begin{array}{r|rrrr} 1 & 2 & 0 & -1 & -1 \\ & & 2 & 2 & 1 \\ \hline & 2 & 2 & 1 & 0 \end{array}$$

$$2x^3 - x - 1 = (x - 1) (2x^2 + 2x + 1)$$

BACK INTO THE SUM

$$= \lim_{x \rightarrow 1} \frac{(x - 1) (3x^2 + 7x + 1)}{(x - 1) (2x^2 + 2x + 1)} \quad \text{CUT}$$

$$(x \rightarrow 1 ; x \neq 1 \therefore x - 1 \neq 0)$$

$$= \lim_{x \rightarrow 1} \frac{3x^2 + 7x + 1}{2x^2 + 2x + 1} \quad \text{COPY}$$

$$= \frac{3(1)^2 + 7(1) + 1}{2(1)^2 + 2(1) + 1} \quad \text{PASTE}$$

$$= \frac{3 + 7 + 1}{2 + 2 + 1}$$

$$= \frac{11}{5}$$

$$05. \lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 4}{x^3 - 2x^2 - 4x + 8}$$

$$x^3 - 3x^2 + 4 = (x - 2) (?)$$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & 0 & 4 \\ & & 2 & -2 & -4 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$$x^3 - 3x^2 + 4 = (x - 2) (x^2 - x - 2)$$

$$x^3 - 2x^2 - 4x + 8 = (x - 2) (?)$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -4 & 8 \\ & & 2 & 0 & -8 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$$x^3 - 2x^2 - 4x + 8 = (x - 2) (x^2 - 4)$$

BACK INTO THE SUM

$$= \lim_{x \rightarrow 2} \frac{(x - 2) (x^2 - x - 2)}{(x - 2) (x^2 - 4)} \quad \text{CUT}$$

$$(x \rightarrow 2 ; x \neq 2 \therefore x - 2 \neq 0)$$

$$= \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 4} \quad \text{COPY}$$

$$= \lim_{x \rightarrow 2} \frac{x^2 - 2x + x - 2}{(x - 2)(x + 2)}$$

$$= \lim_{x \rightarrow 2} \frac{x(x - 2) + 1(x - 2)}{(x - 2)(x + 2)}$$

$$= \lim_{x \rightarrow 2} \frac{(x - 2)(x + 1)}{(x - 2)(x + 2)} \quad \text{CUT}$$

$$= \lim_{x \rightarrow 2} \frac{x + 1}{x + 2} \quad \text{COPY}$$

$$= \frac{2 + 1}{2 + 2} \quad \text{PASTE}$$

$$= \frac{3}{4}$$

$$06. \quad \lim_{x \rightarrow 1} \frac{x^3 - x^2 - x + 1}{x^3 + x^2 - 5x + 3}$$

$$x^3 - x^2 - x + 1 = (x - 1) (?)$$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -1 & 1 \\ & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$$x^3 - x^2 - x + 1 = (x - 1)(x^2 - 1)$$

$$x^3 + x^2 - 5x + 3 = (x - 1) (?)$$

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -5 & 3 \\ & & 1 & 2 & -3 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$$\begin{aligned} x^3 + x^2 - 5x + 3 \\ = (x - 1) (x^2 + 2x - 3) \end{aligned}$$

BACK INTO THE SUM

$$= \lim_{x \rightarrow 1} \frac{(x-1)(x^2-1)}{(x-1)(x^2+2x-3)}$$

$$(x \rightarrow 1 ; x \neq 1 \therefore x - 1 \neq 0)$$

$$= \lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + 2x - 3}$$

$$= \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{x^2 + 3x - x - 3}$$

$$= \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{x(x+3) - 1(x+3)}$$

$$= \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x-1)(x+3)}$$

$$= \lim_{x \rightarrow 1} \frac{x+1}{x+3}$$

$$= \frac{1+1}{1+3} = \frac{1}{2}$$

$$07. \quad \lim_{x \rightarrow 3} \frac{x^3 - 3x^2 - 9x + 27}{x^3 - 6x^2 - 9x}$$

$$x^3 - 3x^2 - 9x + 27 = (x - 3) (?)$$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & -9 & 27 \\ & & 3 & 0 & -27 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$$x^3 - 3x^2 - 9x + 27 = (x - 3) (x^2 - 9)$$

$$x^3 - 6x^2 - 9x = (x - 3) (?)$$

$$\begin{array}{r|rrrr} 3 & 1 & -6 & 9 & 0 \\ & & 3 & -9 & 0 \\ \hline & 1 & -3 & 0 & 0 \end{array}$$

$$x^3 - 6x^2 - 9x = (x - 3) (x^2 - 3x)$$

BACK INTO THE SUM

$$= \lim_{x \rightarrow 3} \frac{(x-3)(x^2-9)}{(x-3)(x^2-3x)}$$

$$(x \rightarrow 3 ; x \neq 3 \therefore x - 3 \neq 0)$$

$$= \lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 3x}$$

$$= \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{x(x-3)}$$

$$= \lim_{x \rightarrow 3} \frac{x+3}{x}$$

$$= \frac{3+3}{3}$$

$$= 2$$

CUT

COPY

CUT

COPY

CUT

COPY

CUT

COPY

PASTE

SOLUTION TO Q SET - 4

$$01. \quad \lim_{x \rightarrow a} \frac{x^{25} - a^{25}}{x^{15} - a^{15}}$$

Divide numerator and denominator by $x - a$

$$x \rightarrow a ; x \neq a \quad \therefore x - a \neq 0$$

$$= \lim_{x \rightarrow a} \frac{\frac{x^{25} - a^{25}}{x - a}}{\frac{x^{15} - a^{15}}{x - a}}$$

$$= \frac{25 a^{25-1}}{15 a^{15-1}}$$

$$= \frac{25(a)^{24}}{15(a)^{14}}$$

$$= \frac{5(a)^{24-14}}{3}$$

$$= \frac{5a^{10}}{3}$$

$$02. \quad \lim_{x \rightarrow a} \frac{x^7 - a^7}{x^{11} - a^{11}}$$

Divide numerator and denominator by $x - a$

$$= \lim_{x \rightarrow a} \frac{\frac{x^7 - a^7}{x - a}}{\frac{x^{11} - a^{11}}{x - a}}$$

$$= \frac{7 a^{7-1}}{11 a^{11-1}}$$

$$= \frac{7 a^6}{11 a^{10}}$$

$$= \frac{7}{11 a^4}$$

$$03. \quad \lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16}$$

$$= \lim_{x \rightarrow 4} \frac{x^3 - 4^3}{x^2 - 4^2}$$

Divide numerator and denominator by

$$x - 4, x \rightarrow 4 ; x \neq 4 \quad \therefore x - 4 \neq 0$$

$$= \lim_{x \rightarrow 4} \frac{\frac{x^3 - 4^3}{x - 4}}{\frac{x^2 - 4^2}{x - 4}}$$

$$= \frac{3 \cdot (4)^{3-1}}{2 \cdot (4)^{2-1}}$$

$$= \frac{3 \cdot 4^2}{2 \cdot 4^1}$$

$$= \frac{3 \cdot 4}{2}$$

$$= 6$$

$$04. \quad \lim_{x \rightarrow 2} \frac{x^6 - 64}{x^{10} - 1024}$$

$$= \lim_{x \rightarrow 2} \frac{x^6 - 2^6}{x^{10} - 2^{10}}$$

Divide numerator and denominator by $x - 2$

$$x \rightarrow 2 ; x \neq 2 \quad \therefore x - 2 \neq 0$$

$$= \lim_{x \rightarrow 2} \frac{\frac{x^6 - 2^6}{x - 2}}{\frac{x^{10} - 2^{10}}{x - 2}}$$

$$= \frac{6 \cdot (2)^{6-1}}{10 \cdot (2)^{10-1}}$$

$$= \frac{6 \cdot 2^5}{10 \cdot 2^9}$$

$$= \frac{6}{10 \cdot 2^4}$$

$$= \frac{6}{10 \cdot 16}$$

$$= \frac{3}{80}$$

$$05. \quad \lim_{x \rightarrow a} \frac{x^{-3} - a^{-3}}{x^{-7} - a^{-7}}$$

Divide numerator and denominator by $x - a$

$$x \rightarrow a ; x \neq a \quad \therefore x - a \neq 0$$

$$= \lim_{x \rightarrow a} \frac{\frac{x^{-3} - a^{-3}}{x - a}}{\frac{x^{-7} - a^{-7}}{x - a}}$$

$$= \frac{-3 a^{-3} - 1}{-7 a^{-7} - 1}$$

$$= \frac{3 a^{-4}}{7 a^{-8}}$$

$$= \frac{3 a^{-4+8}}{7}$$

$$= \frac{3 a^4}{7}$$

$$06. \quad \lim_{x \rightarrow 3} \frac{x^{-4} - 3^{-4}}{x^{-7} - 3^{-7}}$$

Divide numerator and denominator by $x - 3$

$$x \rightarrow 3 ; x \neq 3 \quad \therefore x - 3 \neq 0$$

$$= \lim_{x \rightarrow 3} \frac{\frac{x^{-4} - 3^{-4}}{x - 3}}{\frac{x^{-7} - 3^{-7}}{x - 3}}$$

$$= \frac{-4 \cdot 3^{-4} - 1}{-7 \cdot 3^{-7} - 1}$$

$$= \frac{4 \cdot 3^{-5}}{7 \cdot 3^{-8}}$$

$$= \frac{4 \cdot 3^{-5+8}}{7}$$

$$= \frac{4 \cdot (3)^3}{7}$$

$$= \frac{108}{7}$$

$$07. \quad \lim_{x \rightarrow 3} \frac{x^{1/4} - 3^{1/4}}{x^{1/3} - 3^{1/3}}$$

Divide numerator and denominator by $x - 3$

$$x \rightarrow 3 ; x \neq 3 \quad \therefore x - 3 \neq 0$$

$$= \lim_{x \rightarrow 3} \frac{\frac{x^{1/4} - 3^{1/4}}{x - 3}}{\frac{x^{1/3} - 3^{1/3}}{x - 3}}$$

$$= \frac{\frac{1}{4} \cdot 3^{1/4} - 1}{\frac{1}{3} \cdot 3^{1/3} - 1}$$

$$= \frac{3 \cdot 3^{1/4} - 1 - 1/3 + 1}{4}$$

$$= \frac{3 \cdot 3^{1/4} - 1/3}{4}$$

$$= \frac{3(3)^{-1/12}}{4}$$

$$= \frac{3^{1-1/12}}{4}$$

$$= \frac{3^{-11/12}}{4}$$

08. $\lim_{x \rightarrow k} \frac{x^8 - k^8}{x - k} = 8$, find k

$$8k^{(8-1)} = 8$$

$$8k^7 = 8$$

$$k^7 = 1$$

$$k = 1$$

09. $\lim_{x \rightarrow k} \frac{x^5 - k^5}{x - k} = 80$, find k

$$5k^{(5-1)} = 80$$

$$5k^4 = 80$$

$$k^4 = 16$$

$$k^2 = 4$$

$$k = \pm 2$$

10. $\lim_{x \rightarrow 1} \frac{x + x^2 + x^3 - 3}{x - 1}$

Solution

$$= \lim_{x \rightarrow 1} \frac{x + x^2 + x^3 - 3}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{x - 1 + x^2 - 1 + x^3 - 1}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{x - 1 + x^2 - 1^2 + x^3 - 1^3}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{x - 1 + \frac{x^2 - 1^2}{x - 1} + \frac{x^3 - 1^3}{x - 1}}$$

$$= 1 + 2(1)^{2-1} + 3(1)^{3-1}$$

$$= 1 + 2(1)^1 + 3(1)^2$$

$$= 1 + 2(1) + 3(1)$$

$$= 6$$

11. $\lim_{x \rightarrow 3} \frac{x + x^2 + x^3 - 39}{x - 3}$

$$= \lim_{x \rightarrow 3} \frac{x + x^2 + x^3 - 39}{x - 3}$$

$$= \lim_{x \rightarrow 3} \frac{x - 3 + x^2 - 9 + x^3 - 27}{x - 3}$$

$$= \lim_{x \rightarrow 3} \frac{x - 3 + \frac{x^2 - 3^2}{x - 3} + \frac{x^3 - 3^3}{x - 3}}$$

$$= \lim_{x \rightarrow 3} \frac{x - 3 + \frac{x^2 - 3^2}{x - 3} + \frac{x^3 - 3^3}{x - 3}}$$

$$= 1 + 2(3)^{2-1} + 3(3)^{3-1}$$

$$= 1 + 2(3)^1 + 3(3)^2$$

$$= 1 + 2(3) + 3(9)$$

$$= 1 + 6 + 27$$

$$= 34$$

LIMITS OF TRIGNOMETRIC FUNCTIONS

Q SET - 1

01. $\lim_{x \rightarrow 0} \frac{\sin x \cdot \sin 3x}{2x^2}$ ans : $3/2$

11. $\lim_{x \rightarrow 0} \frac{x^2}{\cos 3x - \cos 9x}$ ans : $1/36$

02. $\lim_{x \rightarrow 0} \frac{\sin 3x \cdot \sin 5x}{8x^2}$ ans : $15/8$

12. $\lim_{x \rightarrow 0} \frac{\cos 4x - \cos 8x}{x \cdot \tan x}$ ans : 24

03. $\lim_{x \rightarrow 0} \frac{\sin 3x - \sin 2x}{x}$ ans : 1

13. $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$ ans : 2

04. $\lim_{x \rightarrow 0} \frac{\sin 6x - \sin 4x}{x}$ ans : 2

14. $\lim_{x \rightarrow 0} \frac{\sin x \cdot (1 - \cos x)}{x^3}$ ans : $1/2$

05. $\lim_{x \rightarrow 0} \frac{5\sin x - x \cdot \cos x}{2\tan x + x^2}$ ans : 2

15. $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$ ans : $1/2$

Q SET - 2

06. $\lim_{x \rightarrow 0} \frac{x \cos x + \sin x}{x^2 + \tan x}$ ans : 2

01. $\lim_{x \rightarrow \pi/2} \frac{\cot^2 x}{1 - \sin x}$ ans : 1

07. $\lim_{x \rightarrow 0} \frac{7x \cos x + 3 \sin x}{3x^2 + \tan x}$ ans : 10

02. $\lim_{x \rightarrow 0} \frac{\tan^2 x}{1 - \cos x}$ ans : 2

08. $\lim_{x \rightarrow 0} \frac{4 \sin x - 3 \tan x}{2x + 3 \sin x}$ ans : $1/5$

03. $\lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{3 + \cos x} - 2}$ ans : -8

09. $\lim_{x \rightarrow 0} \frac{\cos 3x - \cos 7x}{x^2}$ ans : 20

04. $\lim_{x \rightarrow \pi/2} \frac{\cos^2 x}{\sqrt{3 + \sin x} - 2}$ ans : -8

10. $\lim_{x \rightarrow 0} \frac{\cos 5x - \cos 11x}{x^2}$ ans : 48

05. $\lim_{x \rightarrow \pi/2} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x}$ ans : $1/4\sqrt{2}$

$$06. \quad \lim_{x \rightarrow \pi} \frac{\sqrt{1 - \cos x} - \sqrt{2}}{\sin^2 x} \quad \text{ans : } 1/4\sqrt{2}$$

$$02. \quad \lim_{x \rightarrow \pi} \frac{\sqrt{5 + \cos x} - 2}{(\pi - x)^2} \quad \text{ans : } 1/8$$

$$07. \quad \lim_{x \rightarrow 0} \frac{\sin^2 x}{1 - \cos^3 x} \quad \text{ans : } 2/3$$

$$03. \quad \lim_{x \rightarrow \pi/4} \frac{1 - \tan x}{\pi - 4x} \quad \text{ans : } 1/2$$

$$08. \quad \lim_{x \rightarrow \pi} \frac{\sin^2 x}{1 + \cos^3 x} \quad \text{ans : } 2/3$$

$$04. \quad \lim_{x \rightarrow \pi/3} \frac{\sqrt{3} - \tan x}{\pi - 3x} \quad \text{ans : } 4/3$$

$$09. \quad \lim_{x \rightarrow \pi/2} \frac{1 - \sin^3 x}{\cos^2 x} \quad \text{ans : } 3/2$$

$$05. \quad \lim_{x \rightarrow \pi/4} \frac{\cos x - \sin x}{\pi - 4x} \quad \text{ans : } \sqrt{2}/4$$

$$10. \quad \lim_{x \rightarrow \pi/4} \frac{1 - \tan^3 x}{\sec^2 x - 2} \quad \text{ans : } -3/2$$

$$06. \quad \lim_{x \rightarrow 1} \frac{1 + \cos \pi x}{(1 - x)^2} \quad \text{ans : } \pi^2/2$$

$$11. \quad \lim_{x \rightarrow \pi/4} \frac{\operatorname{cosec}^2 x - 2}{1 - \cot^3 x} \quad \text{ans : } -2/3$$

$$07. \quad \lim_{x \rightarrow \pi/2} \frac{\operatorname{cosec} x - 1}{(\pi/2 - x)^2} \quad \text{ans : } 1/2$$

$$12. \quad \lim_{x \rightarrow \pi/4} \frac{\cot x - 1}{2 - \operatorname{cosec}^2 x} \quad \text{ans : } -1/2$$

$$08. \quad \lim_{x \rightarrow \pi/2} \frac{3 \cos x + \cos 3x}{(\pi - 2x)^3} \quad \text{ans : } 1/2$$

$$14. \quad \lim_{x \rightarrow \pi/6} \frac{2 - \operatorname{cosec} x}{\cot^2 x - 3} \quad \text{ans : } -1/4$$

$$09. \quad \lim_{x \rightarrow \pi/2} \frac{\sec x - \tan x}{\pi/2 - x} \quad \text{ans : } 1/2$$

$$15. \quad \lim_{x \rightarrow \pi/3} \frac{\sec^3 x - 8}{\tan^2 x - 3} \quad \text{ans : } 3$$

Q SET - 3

$$01. \quad \lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} \quad \text{ans : } 1/4$$

SOLUTION TO Q SET - 1

01.

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sin x \cdot \sin 3x}{2x^2} \\ = & \lim_{x \rightarrow 0} \frac{1}{2} \frac{\sin x \cdot \sin 3x}{x^2} && \text{SIDE} \\ = & \lim_{x \rightarrow 0} \frac{1}{2} \frac{\sin x}{x} \cdot \frac{\sin 3x}{x} && \text{DISITRIBUTE} \\ = & \lim_{x \rightarrow 0} \frac{1}{2} \frac{\sin x}{x} \cdot 3 \frac{\sin 3x}{3x} \\ = & \frac{1}{2} (1) \cdot 3 \cdot (1) && \text{KAHI FORMULA} \\ & && \text{KAHI PASTE} \\ = & \frac{3}{2} \end{aligned}$$

02.

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sin 3x \cdot \sin 5x}{8x^2} \\ = & \lim_{x \rightarrow 0} \frac{1}{8} \frac{\sin 3x \cdot \sin 5x}{x^2} && \text{SIDE} \\ = & \lim_{x \rightarrow 0} \frac{1}{2} \frac{\sin 3x}{x} \cdot \frac{\sin 5x}{x} && \text{DISITRIBUTE} \\ = & \lim_{x \rightarrow 0} \frac{1}{2} \frac{3 \sin 3x}{3x} \cdot \frac{5 \sin 5x}{5x} \\ = & \frac{1}{2} 3(1) \cdot 5 \cdot (1) && \text{KAHI FORMULA} \\ & && \text{KAHI PASTE} \\ = & \frac{15}{2} \end{aligned}$$

03.

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sin 3x - \sin 2x}{x} \\ = & \lim_{x \rightarrow 0} \frac{\sin 3x}{x} - \frac{\sin 2x}{x} && \text{DISITRIBUTE} \\ = & \lim_{x \rightarrow 0} 3 \frac{\sin 3x}{3x} - 2 \frac{\sin 2x}{2x} \\ = & 3(1) - 2 \cdot (1) && \text{KAHI FORMULA} \\ & && \text{KAHI PASTE} \\ = & 1 \end{aligned}$$

04.

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sin 6x - \sin 4x}{x} \\ = & \lim_{x \rightarrow 0} \frac{\sin 6x}{x} - \frac{\sin 4x}{x} && \text{DISITRIBUTE} \\ = & \lim_{x \rightarrow 0} 6 \frac{\sin 6x}{6x} - 4 \frac{\sin 4x}{4x} \\ = & 6(1) - 4 \cdot (1) && \text{KAHI FORMULA} \\ & && \text{KAHI PASTE} \\ = & 2 \end{aligned}$$

05.

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{5\sin x - x \cdot \cos x}{2\tan x + x^2} \\ & \text{Divide Numerator \& Denominator by } x, \\ & x \rightarrow 0, x \neq 0 \\ = & \lim_{x \rightarrow 0} \frac{\frac{5\sin x - x \cdot \cos x}{x}}{\frac{2\tan x + x^2}{x}} \end{aligned}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{5 \sin x - \cancel{x} \cos x}{x}}{\frac{2 \tan x + \cancel{x^2}}{x}} \quad \text{DISITRIBUTE}$$

$$= \lim_{x \rightarrow 0} \frac{5 \sin x - \cos x}{2 \frac{\tan x}{x} + x} \quad \text{CUT}$$

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PASTE HOTI

$$= \frac{5(1) - \cos 0}{2(1) + 0}$$

$$= \frac{5 - 1}{2}$$

$$= 2$$

06.

$$\lim_{x \rightarrow 0} \frac{x \cos x + \sin x}{x^2 + \tan x}$$

Divide Numerator & Denominator by x ,
 $x \rightarrow 0 , x \neq 0$

$$= \lim_{x \rightarrow 0} \frac{\frac{x \cos x + \sin x}{x}}{\frac{x^2 + \tan x}{x}}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\cancel{x} \cos x + \sin x}{x}}{\frac{\cancel{x^2} + \tan x}{x}} \quad \text{DISITRIBUTE}$$

$$= \lim_{x \rightarrow 0} \frac{\cos x + \frac{\sin x}{x}}{x + \frac{\tan x}{x}} \quad \text{CUT}$$

07.

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$$= \frac{\cos 0 + 1}{0 + 1}$$

$$= 1 + 1$$

$$= 2$$

07.

$$\lim_{x \rightarrow 0} \frac{7x \cos x + 3 \sin x}{3x^2 + \tan x}$$

Divide Numerator & Denominator by x ,
 $x \rightarrow 0 , x \neq 0$

$$= \lim_{x \rightarrow 0} \frac{\frac{7x \cos x + 3 \sin x}{x}}{\frac{3x^2 + \tan x}{x}}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\cancel{7x} \cos x + 3 \sin x}{x}}{\frac{\cancel{3x^2} + \tan x}{x}} \quad \text{DISITRIBUTE}$$

$$= \lim_{x \rightarrow 0} \frac{7 \cos x + 3 \frac{\sin x}{x}}{3x + \frac{\tan x}{x}} \quad \text{CUT}$$

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$$= \frac{7 \cos 0 + 3(1)}{3(0) + 1}$$

$$= 7 + 3$$

$$= 10$$

08.

$$\lim_{x \rightarrow 0} \frac{4 \sin x - 3 \tan x}{2x + 3 \sin x}$$

Divide Numerator & Denominator by x ,
 $x \rightarrow 0, x \neq 0$

$$= \lim_{x \rightarrow 0} \frac{\frac{4 \sin x - 3 \tan x}{x}}{\frac{2x + 3 \sin x}{x}}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{4 \sin x}{x} - 3 \frac{\tan x}{x}}{\frac{2x}{x} + 3 \frac{\sin x}{x}} \quad \text{DISITRIBUTE}$$

CUT

$$= \lim_{x \rightarrow 0} \frac{\frac{4 \sin x}{x} - 3 \frac{\tan x}{x}}{2 + 3 \frac{\sin x}{x}} \quad \text{COPY}$$

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$$= \frac{4(1) - 3(1)}{2 + 3(1)}$$

$$= \frac{1}{5}$$

$$= 10$$

09.

$$\lim_{x \rightarrow 0} \frac{\cos 3x - \cos 7x}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{-2 \sin \left(\frac{3x + 7x}{2} \right) \cdot \sin \left(\frac{3x - 7x}{2} \right)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{-2 \sin 5x \cdot \sin (-2x)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{2 \cdot \sin 5x \cdot \sin 2x}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{2 \cdot \frac{\sin 5x}{x} \cdot \frac{\sin 2x}{x}}{x} \quad \text{DISITRIBUTE}$$

$$= \lim_{x \rightarrow 0} \frac{2 \cdot 5 \frac{\sin 5x}{5x} \cdot 2 \frac{\sin 2x}{2x}}{x}$$

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 PASTE HOTI

$$= 2 \cdot 5(1) \cdot 2(1)$$

$$= 20$$

10.

$$\lim_{x \rightarrow 0} \frac{\cos 5x - \cos 11x}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{-2 \sin \left(\frac{5x + 11x}{2} \right) \cdot \sin \left(\frac{5x - 11x}{2} \right)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{-2 \sin 8x \cdot \sin (-3x)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{2 \cdot \sin 8x \cdot \sin 3x}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{2 \cdot \frac{\sin 8x}{x} \cdot \frac{\sin 3x}{x}}{x} \quad \text{DISITRIBUTE}$$

$$= \lim_{x \rightarrow 0} \frac{2 \cdot 8 \frac{\sin 8x}{8x} \cdot 3 \frac{\sin 3x}{3x}}{x}$$

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 PASTE HOTI

$$= 2 \cdot 8(1) \cdot 3(1) = 48$$

$$\begin{aligned}
11. \quad & \lim_{x \rightarrow 0} \frac{x^2}{\cos 3x - \cos 9x} \\
= & \lim_{x \rightarrow 0} \frac{x^2}{-2 \sin \left(\frac{3x + 9x}{2} \right) \cdot \sin \left(\frac{3x - 9x}{2} \right)} \\
= & \lim_{x \rightarrow 0} \frac{x^2}{-2 \sin 6x \cdot \sin (-3x)} \\
= & \lim_{x \rightarrow 0} \frac{x^2}{2 \sin 6x \cdot \sin 3x} \\
= & \lim_{x \rightarrow 0} \frac{x}{2 \sin 6x} \cdot \frac{x}{\sin 3x} \\
= & \lim_{x \rightarrow 0} \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{6x}{\sin 6x} \cdot \frac{1}{3} \cdot \frac{3x}{\sin 3x} \\
= & \frac{1}{2} \cdot \frac{1}{6} \cdot (1) \cdot \frac{1}{3} \cdot (1) \\
= & \frac{1}{36}
\end{aligned}$$

$$\begin{aligned}
12. \quad & \lim_{x \rightarrow 0} \frac{\cos 4x - \cos 8x}{x \cdot \tan x} \\
= & \lim_{x \rightarrow 0} \frac{-2 \sin \left(\frac{4x + 8x}{2} \right) \cdot \sin \left(\frac{4x - 8x}{2} \right)}{x \cdot \tan x} \\
= & \lim_{x \rightarrow 0} \frac{-2 \sin 6x \cdot \sin (-2x)}{x \cdot \tan x} \\
= & \lim_{x \rightarrow 0} \frac{2 \cdot \sin 6x \cdot \sin 2x}{x \cdot \tan x}
\end{aligned}$$

Divide Numerator & Denominator by x^2 ,
 $x \rightarrow 0$, $x \neq 0$, $x^2 \neq 0$

$$\begin{aligned}
& = \lim_{x \rightarrow 0} \frac{\frac{2 \cdot \sin 6x \cdot \sin 2x}{x^2}}{\frac{x \tan x}{x^2}} \\
& = \lim_{x \rightarrow 0} \frac{2 \cdot \frac{\sin 6x}{x} \cdot \frac{\sin 2x}{x}}{\cancel{x} \frac{\tan x}{x^2}} \quad \text{DISTRIBUTE} \\
& \quad \text{CUT} \\
& = \lim_{x \rightarrow 0} \frac{2 \cdot 6 \frac{\sin 6x}{6x} \cdot 2 \frac{\sin 2x}{2x}}{\frac{\tan x}{x}} \\
& \quad \text{LIMIT JATHI VALUE AATI , KAIHI FORMULA LAGTI KAIHI} \\
& \quad \text{PASTE HOTI} \\
& = \frac{2 \cdot 6(1) \cdot 2(1)}{1} = 24
\end{aligned}$$

$$\begin{aligned}
13. \quad & \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} \\
= & \lim_{x \rightarrow 0} \frac{2 \cdot \sin^2 x}{x^2} \\
= & \lim_{x \rightarrow 0} 2 \left(\frac{\sin x}{x} \right)^2 \quad \text{SQUARE-SQUARE} \\
& \quad \text{THE WHOLE SQUARE} \\
= & 2(1)^2 \\
= & 2
\end{aligned}$$

14.

$$\lim_{x \rightarrow 0} \frac{\sin x \cdot (1 - \cos x)}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x \cdot 2 \sin^2 (x/2)}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{2 \sin^2 (x/2)}{x^2} \quad \text{DISTRIBUTE}$$

SQUARE-SQUARE THE WHOLE SQUARE

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot 2 \left(\frac{\sin (x/2)}{x} \right)^2$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot 2 \left(\frac{1}{2} \frac{\sin (x/2)}{x/2} \right)^2$$

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$$= 1 \cdot 2 \left(\frac{1}{2} (1) \right)^2 = \frac{1}{2}$$

15.

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\sin x - \sin x \cdot \cos x}{\cos x}}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x (1 - \cos x)}{\cos x \cdot x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\tan x (1 - \cos x)}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\tan x \cdot 2 \sin^2 (x/2)}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\tan x}{x} \cdot \frac{2 \sin^2 (x/2)}{x^2} \quad \text{DISTRIBUTE}$$

SQUARE-SQUARE THE WHOLE SQUARE

$$= \lim_{x \rightarrow 0} \frac{\tan x}{x} \cdot 2 \left(\frac{\sin (x/2)}{x} \right)^2$$

$$= \lim_{x \rightarrow 0} \frac{\tan x}{x} \cdot 2 \left(\frac{1}{2} \frac{\sin (x/2)}{x/2} \right)^2$$

LIMIT JATHI VALUE AATI , KAHİ FORMULA LAGTI KAIHI

PASTE HOTI

$$= 1 \cdot 2 \left(\frac{1}{2} (1) \right)^2$$

$$= \frac{1}{2}$$

SOLUTION TO Q SET - 2

01.

$$\begin{aligned} & \lim_{x \rightarrow \pi/2} \frac{\cot^2 x}{1 - \sin x} \\ &= \lim_{x \rightarrow \pi/2} \frac{\cos^2 x}{\sin^2 x \cdot (1 - \sin x)} \\ &= \lim_{x \rightarrow \pi/2} \frac{1 - \sin^2 x}{\sin^2 x \cdot (1 - \sin x)} \\ &= \lim_{x \rightarrow \pi/2} \frac{\cancel{(1 - \sin x)}(1 + \sin x)}{\sin^2 x \cdot \cancel{(1 - \sin x)}} \quad \text{CUT} \end{aligned}$$

$$x \rightarrow \pi/2 ; \sin x \rightarrow \sin \pi/2 ; \sin x \rightarrow 1 , \\ \sin x \neq 1 ; 1 - \sin x \neq 0$$

$$\begin{aligned} &= \lim_{x \rightarrow \pi/2} \frac{1 + \sin x}{\sin^2 x} \quad \text{COPY} \\ &= \frac{1 + \sin \pi/2}{\sin^2 \pi/2} \quad \text{PASTE} \\ &= \frac{1 + 1}{1} = 2 \end{aligned}$$

02.

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\tan^2 x}{1 - \cos x} \\ &= \lim_{x \rightarrow 0} \frac{\sin^2 x}{\cos^2 x \cdot (1 - \cos x)} \\ &= \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{\cos^2 x \cdot (1 - \cos x)} \\ &= \lim_{x \rightarrow 0} \frac{\cancel{(1 - \cos x)}(1 + \cos x)}{\cos^2 x \cdot \cancel{(1 - \cos x)}} \quad \text{CUT} \end{aligned}$$

$$x \rightarrow 0 ; \cos x \rightarrow \cos 0 ; \cos x \rightarrow 1 , \\ \cos x \neq 1 ; 1 - \cos x \neq 0$$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{1 + \cos x}{\cos^2 x} \quad \text{COPY} \\ &= \frac{1 + \cos 0}{\cos^2 0} \quad \text{PASTE} \\ &= \frac{1 + 1}{1} = 2 \end{aligned}$$

03.

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{3 + \cos x} - 2} \\ &= \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{3 + \cos x} - 2} \cdot \frac{\sqrt{3 + \cos x} + 2}{\sqrt{3 + \cos x} + 2} \\ &= \lim_{x \rightarrow 0} \frac{\sin^2 x}{3 + \cos x - 4} \cdot \frac{\sqrt{3 + \cos x} + 2}{1} \\ &= \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{\cos x - 1} \cdot \frac{\sqrt{3 + \cos x} + 2}{1} \\ &= \lim_{x \rightarrow 0} \frac{\cancel{(1 - \cos x)}(1 + \cos x)}{\cancel{(1 - \cos x)}} \cdot \frac{\sqrt{3 + \cos x} + 2}{1} \quad \text{CUT} \end{aligned}$$

$$x \rightarrow 0 ; \cos x \rightarrow \cos 0 ; \cos x \rightarrow 1 , \\ \cos x \neq 1 ; 1 - \cos x \neq 0$$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{1 + \cos x}{-1} \cdot \frac{\sqrt{3 + \cos x} + 2}{1} \quad \text{COPY} \\ &= \lim_{x \rightarrow 0} \frac{1 + \cos 0}{-1} \cdot \frac{\sqrt{3 + \cos 0} + 2}{1} \quad \text{PASTE} \\ &= \frac{1 + 1}{-1} \cdot \frac{\sqrt{3 + 1} + 2}{1} \end{aligned}$$

$$= \frac{2}{-1} \cdot \frac{2+2}{1}$$

$$= -8$$

04.

$$\lim_{x \rightarrow \pi/2} \frac{\cos^2 x}{\sqrt{3 + \sin x} - 2}$$

$$= \lim_{x \rightarrow \pi/2} \frac{\cos^2 x}{\sqrt{3 + \sin x} - 2} \cdot \frac{\sqrt{3 + \sin x} + 2}{\sqrt{3 + \sin x} + 2}$$

$$= \lim_{x \rightarrow \pi/2} \frac{\cos^2 x}{3 + \sin x - 4} \cdot \frac{\sqrt{3 + \sin x} + 2}{1}$$

$$= \lim_{x \rightarrow \pi/2} \frac{1 - \sin^2 x}{\sin x - 1} \cdot \frac{\sqrt{3 + \sin x} + 2}{1}$$

$$= \lim_{x \rightarrow \pi/2} \frac{(1 - \sin x)(1 + \sin x)}{-(1 - \sin x)} \cdot \frac{\sqrt{3 + \sin x} + 2}{1}$$

CUT

$$x \rightarrow \pi/2 ; \sin x \rightarrow \sin \pi/2 ; \sin x \rightarrow 1 ,$$

$$\sin x \neq 1 ; 1 - \sin x \neq 0$$

$$= \lim_{x \rightarrow \pi/2} \frac{1 + \sin x}{-1} \cdot \frac{\sqrt{3 + \sin x} + 2}{1} \quad \text{COPY}$$

$$= \lim_{x \rightarrow \pi/2} \frac{1 + \sin \pi/2}{-1} \cdot \frac{\sqrt{3 + \sin \pi/2} + 2}{1} \quad \text{PASTE}$$

$$= \frac{1+1}{-1} \cdot \frac{\sqrt{3+1}+2}{1}$$

$$= \frac{2}{-1} \cdot \frac{2+2}{1}$$

$$= -8$$

05.

$$\lim_{x \rightarrow \pi/2} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x}$$

$$= \lim_{x \rightarrow \pi/2} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{1 - \sin^2 x} \cdot \frac{\sqrt{2} + \sqrt{1 + \sin x}}{\sqrt{2} + \sqrt{1 + \sin x}}$$

$$= \lim_{x \rightarrow \pi/2} \frac{2 - (1 + \sin x)}{(1 - \sin x)(1 + \sin x)} \cdot \frac{1}{\sqrt{2} + \sqrt{1 + \sin x}}$$

$$= \lim_{x \rightarrow \pi/2} \frac{1 - \sin x}{(1 - \sin x)(1 + \sin x)} \cdot \frac{1}{\sqrt{2} + \sqrt{1 + \sin x}}$$

CUT

$$x \rightarrow \pi/2 ; \sin x \rightarrow \sin \pi/2 ; \sin x \rightarrow 1 ,$$

$$\sin x \neq 1 ; 1 - \sin x \neq 0$$

COPY

$$= \lim_{x \rightarrow \pi/2} \frac{1}{1 + \sin x} \cdot \frac{1}{\sqrt{2} + \sqrt{1 + \sin x}}$$

PASTE

$$= \frac{1}{1 + \sin \pi/2} \cdot \frac{1}{\sqrt{2} + \sqrt{1 + \sin \pi/2}}$$

$$= \frac{1}{1+1} \cdot \frac{1}{\sqrt{2} + \sqrt{1+1}}$$

$$= \frac{1}{2} \cdot \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{4\sqrt{2}}$$

06.

$$\lim_{x \rightarrow \pi} \frac{\sqrt{1 - \cos x} - \sqrt{2}}{\sin^2 x}$$

$$= \lim_{x \rightarrow \pi} \frac{\sqrt{1 - \cos x} - \sqrt{2}}{1 - \cos^2 x} \cdot \frac{\sqrt{1 - \cos x} + \sqrt{2}}{\sqrt{1 - \cos x} + \sqrt{2}}$$

$$= \lim_{x \rightarrow \pi} \frac{1 - \cos x - 2}{1 - \cos^2 x} \frac{1}{\sqrt{1 - \cos x} + \sqrt{2}}$$

$$= \lim_{x \rightarrow \pi} \frac{-\cos x - 1}{(1 + \cos x)(1 - \cos x)} \frac{1}{\sqrt{1 - \cos x} + \sqrt{2}}$$

$$= \lim_{x \rightarrow \pi} \frac{-(1 + \cancel{\cos x})}{(1 + \cancel{\cos x})(1 - \cos x)} \frac{1}{\sqrt{1 - \cos x} + \sqrt{2}}$$

$x \rightarrow \pi$; $\cos x \rightarrow \cos \pi$; $\cos x \rightarrow -1$
 $\cos x \neq -1 \therefore 1 + \cos x \neq 0$

$$= \lim_{x \rightarrow \pi} \frac{1}{1 - \cos x} \frac{1}{\sqrt{1 - \cos x} + \sqrt{2}}$$

$$= \frac{1}{1 - \cos \pi} \frac{1}{\sqrt{1 - \cos \pi} + \sqrt{2}}$$

$$= \frac{1}{1 - (-1)} \frac{1}{\sqrt{1 - (-1)} + \sqrt{2}}$$

$$= \frac{1}{2} \frac{1}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{1}{2} \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{4\sqrt{2}}$$

07.

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{1 - \cos^3 x}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{(1 - \cos x)(1 + \cos x + \cos^2 x)}$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cancel{\cos x})(1 + \cos x)}{(1 - \cancel{\cos x})(1 + \cos x + \cos^2 x)} \text{ CUY}$$

$x \rightarrow 0$; $\cos x \rightarrow \cos 0$; $\cos x \rightarrow 1$,
 $\cos x \neq 1$; $1 - \cos x \neq 0$

$$= \lim_{x \rightarrow 0} \frac{1 + \cos x}{1 + \cos x + \cos^2 x} \text{ COPY}$$

$$= \frac{1 + \cos 0}{1 + \cos 0 + \cos^2 0} \text{ PASTE}$$

$$= \frac{1 + 1}{1 + 1 + 1} = \frac{2}{3}$$

08.

$$\lim_{x \rightarrow \pi} \frac{\sin^2 x}{1 + \cos^3 x}$$

$$= \lim_{x \rightarrow \pi} \frac{1 - \cos^2 x}{(1 + \cos x)(1 - \cos x + \cos^2 x)}$$

$$= \lim_{x \rightarrow \pi} \frac{(1 + \cancel{\cos x})(1 - \cos x)}{(1 + \cancel{\cos x})(1 - \cos x + \cos^2 x)} \text{ CUT}$$

$x \rightarrow \pi$; $\cos x \rightarrow \cos \pi$; $\cos x \rightarrow -1$,
 $\cos x \neq -1$; $1 + \cos x \neq 0$

$$= \lim_{x \rightarrow \pi} \frac{1 - \cos x}{1 - \cos x + \cos^2 x} \text{ COPY}$$

$$= \frac{1 - \cos \pi}{1 - \cos \pi + \cos^2 \pi} \text{ PASTE}$$

$$= \frac{1 + 1}{1 + 1 + 1}$$

$$= \frac{2}{3}$$

09.

$$\lim_{x \rightarrow \pi/2} \frac{1 - \sin^3 x}{\cos^2 x}$$

$$= \lim_{x \rightarrow \pi/2} \frac{(1 - \sin x)(1 + \sin x + \sin^2 x)}{1 - \sin^2 x}$$

$$= \lim_{x \rightarrow \pi/2} \frac{(1 - \cancel{\sin x})(1 + \sin x + \sin^2 x)}{(1 - \cancel{\sin x})(1 + \sin x)}$$

$$x \rightarrow \frac{\pi}{2} ; \sin x \rightarrow \sin \frac{\pi}{2} ; \sin x \rightarrow 1 ,$$

$$\sin x \neq 1 ; 1 - \sin x \neq 0$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \sin x + \sin^2 x}{1 + \sin x}$$

$$= \frac{1 + \sin \frac{\pi}{2} + \sin^2 \frac{\pi}{2}}{1 + \sin \frac{\pi}{2}}$$

$$= \frac{1 + 1 + 1}{1 + 1} = \frac{3}{2}$$

10.

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan^3 x}{\sec^2 x - 2}$$

$1 + \tan^2 x = \sec^2 x$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{(1 - \tan x)(1 + \tan x + \tan^2 x)}{1 + \tan^2 x - 2}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{(1 - \tan x)(1 + \tan x + \tan^2 x)}{\tan^2 x - 1}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{(1 - \tan x)(1 + \tan x + \tan^2 x)}{(\tan x - 1)(\tan x + 1)}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cancel{(1 - \tan x)}(1 + \tan x + \tan^2 x)}{\cancel{-(1 - \tan x)}(\tan x + 1)}$$

CUT

$$x \rightarrow \frac{\pi}{4} ; \tan x \rightarrow \tan \frac{\pi}{4} ; \tan x \rightarrow 1 ,$$

$$\tan x \neq 1 ; 1 - \tan x \neq 0$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 + \tan x + \tan^2 x}{-(\tan x + 1)} \quad \text{COPY}$$

$$= \frac{1 + \tan \frac{\pi}{4} + \tan^2 \frac{\pi}{4}}{-(\tan \frac{\pi}{4} + 1)} \quad \text{PASTE}$$

$$= \frac{1 + 1 + 1}{-(1 + 1)}$$

$$= -3/2$$

11.

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{1 - \cot^3 x}$$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 + \cot^2 x - 2}{(1 - \cot x)(1 + \cot x + \cot^2 x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cot^2 x - 1}{(1 - \cot x)(1 + \cot x + \cot^2 x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cot x - 1)(\cot x + 1)}{(1 - \cot x)(1 + \cot x + \cot^2 x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cancel{-(1 - \cot x)}(\cot x + 1)}{\cancel{(1 - \cot x)}(1 + \cot x + \cot^2 x)}$$

$$x \rightarrow \frac{\pi}{4} ; \cot x \rightarrow \cot \frac{\pi}{4} ; \cot x \rightarrow 1 ,$$

$$\cot x \neq 1 ; 1 - \cot x \neq 0$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{-(\cot x + 1)}{1 + \cot x + \cot^2 x} \quad \text{COPY}$$

$$= \frac{-(\cot \frac{\pi}{4} + 1)}{1 + \cot \frac{\pi}{4} + \cot^2 \frac{\pi}{4}} \quad \text{PASTE}$$

$$= \frac{-(1 + 1)}{1 + 1 + 1}$$

$$= -2/3$$

12.

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cot x - 1}{2 - \operatorname{cosec}^2 x}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cot x - 1}{2 - (1 + \cot^2 x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cot x - 1}{2 - 1 - \cot^2 x}$$

$$= \lim_{x \rightarrow \pi/4} \frac{\cot x - 1}{1 - \cot^2 x} \quad x \rightarrow \pi/6 ; \sin x \rightarrow \sin \pi/6 ; \sin x \rightarrow 1/2 ,$$

$$2 \sin x \rightarrow 1 ; 2 \sin x \neq 1 ; 1 - 2 \sin x \neq 0$$

$$= \lim_{x \rightarrow \pi/4} \frac{\cot x - 1}{(1 - \cot x)(1 + \cot x)} \quad = \lim_{x \rightarrow \pi/6} \frac{-1}{1 + 2 \sin x} \quad \text{COPY}$$

$$= \lim_{x \rightarrow \pi/4} \frac{-(1 - \cot x)}{(1 - \cot x)(1 + \cot x)} \quad = \lim_{x \rightarrow \pi/6} \frac{-1}{1 + 2 \sin \pi/6} \quad \text{PASTE}$$

CUT

$$x \rightarrow \pi/4 ; \cot x \rightarrow \cot \pi/4 ; \cot x \rightarrow 1 ,$$

$$\cot x \neq 1 ; 1 - \cot x \neq 0$$

$$= \frac{-1}{1 + 2(1/2)}$$

$$= \lim_{x \rightarrow \pi/4} \frac{-1}{1 + \cot x} \quad \text{COPY}$$

$$= -1/2$$

$$= \frac{-1}{1 + \cot \pi/4} \quad \text{PASTE}$$

14.

$$\lim_{x \rightarrow \pi/6} \frac{2 - \operatorname{cosec} x}{\cot^2 x - 3}$$

$$= \frac{-1}{1 + 1} = \frac{-1}{2}$$

$$= \lim_{x \rightarrow \pi/6} \frac{2 - \operatorname{cosec} x}{\operatorname{cosec}^2 x - 1 - 3}$$

13.

$$\lim_{x \rightarrow \pi/6} \frac{2 \sin x - 1}{4 \cos^2 x - 3}$$

$$= \lim_{x \rightarrow \pi/6} \frac{2 - \operatorname{cosec} x}{\operatorname{cosec}^2 x - 4}$$

$$= \lim_{x \rightarrow \pi/6} \frac{2 \sin x - 1}{4(1 - \sin^2 x) - 3}$$

$$= \lim_{x \rightarrow \pi/6} \frac{2 - \operatorname{cosec} x}{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 2)}$$

$$= \lim_{x \rightarrow \pi/6} \frac{2 \sin x - 1}{4 - 4 \sin^2 x - 3}$$

$$= \lim_{x \rightarrow \pi/6} \frac{-(\operatorname{cosec} x - 2)}{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 2)} \quad \text{CUT}$$

$$= \lim_{x \rightarrow \pi/6} \frac{2 \sin x - 1}{1 - 4 \sin^2 x}$$

$$x \rightarrow \pi/6 ; \operatorname{cosec} x \rightarrow \operatorname{cosec} \pi/6 ;$$

$$\operatorname{cosec} x \rightarrow 2 , \operatorname{cosec} x \neq 2 ; \operatorname{cosec} x - 2 \neq 0$$

$$= \lim_{x \rightarrow \pi/6} \frac{2 \sin x - 1}{(1 - 2 \sin x)(1 + 2 \sin x)}$$

$$= \lim_{x \rightarrow \pi/6} \frac{-1}{\operatorname{cosec} x + 2} \quad \text{COPY}$$

$$= \lim_{x \rightarrow \pi/6} \frac{-(1 - 2 \sin x)}{(1 - 2 \sin x)(1 + 2 \sin x)} \quad \text{CUT}$$

$$= \frac{-1}{\operatorname{cosec} \pi/6 + 2} \quad \text{PASTE}$$

$$= \frac{-1}{2 + 2} = \frac{-1}{4}$$

15.

$$\lim_{x \rightarrow \pi/3} \frac{\sec^3 x - 8}{\tan^2 x - 3}$$

$$= \lim_{x \rightarrow \pi/3} \frac{\sec^3 x - 2^3}{\tan^2 x - 3}$$

$$= \lim_{x \rightarrow \pi/3} \frac{(\sec x - 2)(\sec^2 x + 2\sec x + 4)}{\sec^2 x - 1 - 3}$$

$$= \lim_{x \rightarrow \pi/3} \frac{(\sec x - 2)(\sec^2 x + 2\sec x + 4)}{\sec^2 x - 4}$$

$$= \lim_{x \rightarrow \pi/3} \frac{(\sec x - 2)(\sec^2 x + 2\sec x + 4)}{(\sec x - 2)(\sec x + 2)}$$

$$x \rightarrow \pi/3 ; \sec x \rightarrow \sec \pi/3 ; \sec x \rightarrow 2 , \\ \sec x \neq 2 ; \sec x - 2 \neq 0$$

$$= \frac{\sec^2 \pi/3 + 2\sec \pi/3 + 4}{\sec \pi/3 + 2}$$

$$= \frac{2^2 + 2(2) + 4}{2 + 2}$$

$$= 3$$

SOLUTION TO Q SET - 3

01.

$$\lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$$

$$= \lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} \cdot \frac{\sqrt{2 + \cos x} + 1}{\sqrt{2 + \cos x} + 1}$$

$$= \lim_{x \rightarrow \pi} \frac{2 + \cos x - 1}{(\pi - x)^2} \cdot \frac{1}{\sqrt{2 + \cos x} + 1}$$

$$= \lim_{x \rightarrow \pi} \frac{1 + \cos x}{(\pi - x)^2} \cdot \frac{1}{\sqrt{2 + \cos x} + 1}$$

Put $x = \pi + h$

$$= \lim_{h \rightarrow 0} \frac{1 + \cos(\pi + h)}{[(\pi - (\pi + h))]^2} \cdot \frac{1}{\sqrt{2 + \cos(\pi + h)} + 1}$$

$$= \lim_{h \rightarrow 0} \frac{1 - \cos h}{(\pi - \pi - h)^2} \cdot \frac{1}{\sqrt{2 - \cos h} + 1}$$

$$= \lim_{h \rightarrow 0} \frac{1 - \cos h}{h^2} \cdot \frac{1}{\sqrt{2 - \cos h} + 1}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin^2(h/2)}{h^2} \cdot \frac{1}{\sqrt{2 - \cos h} + 1}$$

$$= \lim_{h \rightarrow 0} 2 \left(\frac{1 \sin(h/2)}{2 \cdot h/2} \right)^2 \cdot \frac{1}{\sqrt{2 - \cos h} + 1}$$

$$= \lim_{h \rightarrow 0} 2 \left(\frac{1}{2} (1) \right)^2 \cdot \frac{1}{\sqrt{2 - \cos 0} + 1}$$

$$= 2 \times \frac{1}{4} \cdot \frac{1}{\sqrt{2 - 1} + 2}$$

$$= 2 \times \frac{1}{4} \cdot \frac{1}{1 + 1} = 1/4$$

02.

$$\lim_{x \rightarrow \pi} \frac{\sqrt{5 + \cos x} - 2}{(\pi - x)^2}$$

$$= \lim_{x \rightarrow \pi} \frac{\sqrt{5 + \cos x} - 2}{(\pi - x)^2} \cdot \frac{\sqrt{5 + \cos x} + 2}{\sqrt{5 + \cos x} + 2}$$

$$= \lim_{x \rightarrow \pi} \frac{5 + \cos x - 4}{(\pi - x)^2} \cdot \frac{1}{\sqrt{5 + \cos x} + 2}$$

$$= \lim_{x \rightarrow \pi} \frac{1 + \cos x}{(\pi - x)^2} \cdot \frac{1}{\sqrt{5 + \cos x} + 2}$$

Put $x = \pi + h$

$$= \lim_{h \rightarrow 0} \frac{1 + \cos(\pi + h)}{[(\pi - (\pi + h))]^2} \cdot \frac{1}{\sqrt{5 + \cos(\pi + h)} + 2}$$

$$= \lim_{h \rightarrow 0} \frac{1 - \cos h}{(\pi - \pi - h)^2} \cdot \frac{1}{\sqrt{5 - \cos h} + 2}$$

$$= \lim_{h \rightarrow 0} \frac{1 - \cos h}{h^2} \cdot \frac{1}{\sqrt{5 - \cos h} + 2}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin^2(h/2)}{h^2} \cdot \frac{1}{\sqrt{5 - \cos h} + 2}$$

$$= \lim_{h \rightarrow 0} 2 \left(\frac{1 \sin(h/2)}{2 \cdot h/2} \right)^2 \cdot \frac{1}{\sqrt{5 - \cos h} + 2}$$

$$= \lim_{h \rightarrow 0} 2 \left(\frac{1}{2} (1) \right)^2 \cdot \frac{1}{\sqrt{5 - \cos 0} + 2}$$

$$= 2 \times \frac{1}{4} \cdot \frac{1}{\sqrt{5 - 1} + 2}$$

$$= 2 \times \frac{1}{4} \cdot \frac{1}{2 + 2} = 1/8$$

03.

$$\lim_{x \rightarrow \pi/4} \frac{1 - \tan x}{\pi - 4x}$$

$$\text{PUT } x = \pi/4 + h$$

$$= \lim_{h \rightarrow 0} \frac{1 - \tan(\pi/4 + h)}{\pi - 4(\pi/4 + h)}$$

$$= \lim_{h \rightarrow 0} \frac{1 - \frac{\tan \pi/4 + \tan h}{1 - \tan \pi/4 \cdot \tan h}}{\pi - \pi - 4h}$$

$$= \lim_{h \rightarrow 0} \frac{1 - \frac{1 + \tan h}{1 - \tan h}}{-4h}$$

$$= \lim_{h \rightarrow 0} \frac{1 - \tan h - 1 - \tan h}{-4h \cdot 1 - \tan h}$$

$$= \lim_{h \rightarrow 0} \frac{-2 \tan h}{-4h \cdot 1 - \tan h}$$

$$= \lim_{h \rightarrow 0} \frac{2 \tan h}{4h} \cdot \frac{1}{1 - \tan h}$$

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$$= \frac{2}{4} \cdot \frac{1}{1 - \tan 0}$$

$$= \frac{1}{2}$$

04.

$$\lim_{x \rightarrow \pi/3} \frac{\sqrt{3} - \tan x}{\pi - 3x}$$

$$\text{PUT } x = \pi/3 + h$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3} - \frac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \cdot \tan h}}{\pi - \pi - 3h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3} - \frac{\sqrt{3} + \tan h}{1 - \sqrt{3} \cdot \tan h}}{-3h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3}(1 - \sqrt{3} \cdot \tan h) - \sqrt{3} - \tan h}{-3h \cdot 1 - \sqrt{3} \cdot \tan h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3} - 3 \cdot \tan h - \sqrt{3} - \tan h}{-3h \cdot 1 - \sqrt{3} \cdot \tan h}$$

$$= \lim_{h \rightarrow 0} \frac{-4 \tan h}{-3h \cdot 1 - \sqrt{3} \tan h}$$

$$= \lim_{h \rightarrow 0} \frac{4 \tan h}{3h} \cdot \frac{1}{1 - \sqrt{3} \tan h}$$

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PASTE HOTI

$$= \frac{4}{3} \cdot \frac{1}{1 - \sqrt{3} \cdot \tan 0}$$

$$= \frac{4}{3}$$

05.

$$\lim_{x \rightarrow \pi/4} \frac{\cos x - \sin x}{\pi - 4x}$$

PUT $x = \pi/4 + h$

$$= \lim_{h \rightarrow 0} \frac{\cos(\pi/4 + h) - \sin(\pi/4 + h)}{\pi - 4(\pi/4 + h)}$$

$$= \lim_{h \rightarrow 0} \frac{(\cos \pi/4 \cos h - \sin \pi/4 \cdot \sin h) - (\sin \pi/4 \cosh + \cos \pi/4 \cdot \sin h)}{\pi - \pi - 4h}$$

$$= \lim_{h \rightarrow 0} \frac{(1/\sqrt{2} \cdot \cos h - 1/\sqrt{2} \cdot \sin h) - (1/\sqrt{2} \cdot \cos h + 1/\sqrt{2} \cdot \sin h)}{-4h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{2}} \cosh - \frac{1}{\sqrt{2}} \sinh - \frac{1}{\sqrt{2}} \cosh - \frac{1}{\sqrt{2}} \sinh}{-4h}$$

$$= \lim_{h \rightarrow 0} \frac{-\frac{2}{\sqrt{2}} \sin h}{-4h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2} \cdot \sin h}{4h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2} \cdot \sin h}{4h}$$

$$= \frac{\sqrt{2}}{4}$$

06.

$$\lim_{x \rightarrow 1} \frac{1 + \cos \pi x}{(1 - x)^2}$$

PUT $x = 1 + h$

$$= \lim_{h \rightarrow 0} \frac{1 + \cos \pi(1 + h)}{[1 - (1 + h)]^2}$$

$$= \lim_{h \rightarrow 0} \frac{1 + \cos(\pi + \pi h)}{(1 - 1 - h)^2}$$

$$= \lim_{h \rightarrow 0} \frac{1 - \cos \pi h}{h^2}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{\pi h}{2}}{h^2}$$

$$= \lim_{h \rightarrow 0} 2 \left(\frac{\sin \pi h}{h} \right)^2$$

$$= \lim_{h \rightarrow 0} 2 \left(\frac{\frac{\pi}{2} \sin \frac{\pi h}{2}}{\frac{\pi h}{2}} \right)^2$$

$$= 2 \left(\frac{\pi}{2} (1) \right)^2$$

$$= 2 \frac{\pi^2}{4}$$

$$= \frac{\pi^2}{2}$$

07.

$$\begin{aligned} & \lim_{x \rightarrow \pi/2} \frac{\operatorname{cosec} x - 1}{(\pi/2 - x)^2} \\ &= \lim_{x \rightarrow \pi/2} \frac{\frac{1}{\sin x} - 1}{(\pi/2 - x)^2} \\ &= \lim_{x \rightarrow \pi/2} \frac{1 - \sin x}{(\pi/2 - x)^2 \sin x} \end{aligned}$$

PUT $x = \pi/2 + h$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{1 - \sin(\pi/2 + h)}{[\pi/2 - (\pi/2 + h)]^2 \cdot \sin(\pi/2 + h)} \\ &= \lim_{h \rightarrow 0} \frac{1 - \cos h}{[\pi/2 - \pi/2 - h]^2 \cdot \cos h} \\ &= \lim_{h \rightarrow 0} \frac{1 - \cos h}{h^2 \cdot \cos h} \\ &= \lim_{h \rightarrow 0} \frac{2 \sin^2(h/2)}{h^2} \cdot \frac{1}{\cos h} \\ &= \lim_{h \rightarrow 0} 2 \frac{\sin^2(h/2)}{h^2} \cdot \frac{1}{\cos h} \\ &= \lim_{h \rightarrow 0} 2 \left(\frac{\sin(h/2)}{h} \right)^2 \cdot \frac{1}{\cos h} \\ &= \lim_{h \rightarrow 0} 2 \left(\frac{1}{2} \frac{\sin(h/2)}{h/2} \right)^2 \cdot \frac{1}{\cos h} \\ &= 2 \left(\frac{1}{2} (1) \right)^2 \cdot \frac{1}{\cos 0} \\ &= \frac{1}{2} \end{aligned}$$

08.

$$\begin{aligned} & \lim_{x \rightarrow \pi/2} \frac{3 \cos x + \cos 3x}{(\pi - 2x)^3} \\ &= \lim_{x \rightarrow \pi/2} \frac{3 \cos x + (4 \cos^3 x - 3 \cos x)}{(\pi - 2x)^3} \\ &= \lim_{x \rightarrow \pi/2} \frac{3 \cos x + 4 \cos^3 x - 3 \cos x}{(\pi - 2x)^3} \end{aligned}$$

$$= \lim_{x \rightarrow \pi/2} \frac{4 \cos^3 x}{(\pi - 2x)^3}$$

PUT $x = \pi/2 + h$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{4 \cos^3(\pi/2 + h)}{[\pi - 2(\pi/2 + h)]^3} \\ &= \lim_{h \rightarrow 0} \frac{-4 \sin^3 h}{(\pi - \pi - 2h)^2} \quad \begin{array}{l} \curvearrowright \\ \cos(90 + \theta) = -\sin \theta \end{array} \\ &= \lim_{h \rightarrow 0} \frac{-4 \sin^3 h}{-8h^3} \\ &= \lim_{h \rightarrow 0} \frac{4 \sin^3 h}{8 h^3} \\ &= \lim_{h \rightarrow 0} \frac{4 \left(\frac{\sin h}{h} \right)^3}{8} \\ &= \frac{4 (1)^3}{8} \\ &= \frac{1}{2} \end{aligned}$$

LIMITS OF EXPONENTIAL & LOGARITHMIC FN'S

Q SET - 1

BASED ON $\lim_{x \rightarrow 0} (1+x)^{1/x} = e$

01. $\lim_{x \rightarrow 0} \left(1 + \frac{x}{3}\right)^{\frac{1}{x}}$ ans : $e^{1/3}$

02. $\lim_{x \rightarrow 0} \left(1 + \frac{x}{4}\right)^{\frac{1}{x}}$ ans : $e^{1/4}$

03. $\lim_{x \rightarrow 0} \left(1 + \frac{2x}{3}\right)^{\frac{1}{x}}$ ans : $e^{2/3}$

04. $\lim_{x \rightarrow 0} \left(1 + \frac{3x}{4}\right)^{\frac{1}{x}}$ ans : $e^{3/4}$

05. $\lim_{x \rightarrow 0} (1 + 2x)^{\frac{3}{x}}$ ans : e^6

06. $\lim_{x \rightarrow 0} (1 + 2x)^{\frac{5}{x}}$ ans : e^{10}

07. $\lim_{x \rightarrow 0} \left(\frac{1+4x}{1-4x}\right)^{\frac{1}{x}}$ ans : e^8

08. $\lim_{x \rightarrow 0} \left(\frac{1-5x}{1+5x}\right)^{\frac{1}{x}}$ ans : e^{-10}

09. $\lim_{x \rightarrow 0} \left(\frac{1+8x}{1-8x}\right)^{\frac{1}{x}}$ ans : e^{16}

10. $\lim_{x \rightarrow 0} \left(\frac{1-3x}{1+4x}\right)^{\frac{1}{x}}$ ans : e^{-7}

11. $\lim_{x \rightarrow 0} \left(\frac{4+x}{4-x}\right)^{\frac{1}{x}}$ ans : $e^{1/2}$

Q SET - 2

BASED ON $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$

01. $\lim_{x \rightarrow 0} \frac{\log(1+2x)}{x}$ ans : 2

02. $\lim_{x \rightarrow 0} \frac{\log(1+6x)}{2x}$ ans : 3

03. $\lim_{x \rightarrow 0} \frac{1}{x} \log \left(1 + \frac{8x}{3}\right)$ ans : $8/3$

04. $\lim_{x \rightarrow 0} \frac{\log(1+5x) - \log(1+3x)}{x}$ ans : 2

05. $\lim_{x \rightarrow 0} \frac{\log 7 + \log \left(\frac{x+1}{7}\right)}{x}$ ans : 1

06. $\lim_{x \rightarrow 0} \frac{\log(2+x) - \log 2}{x}$ ans : $1/2$

Q SET - 3

BASED ON $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a$

01. $\lim_{x \rightarrow 0} \frac{5^x - 4^x}{x}$ ans: $\log \left(\frac{5}{4}\right)$

02. $\lim_{x \rightarrow 0} \frac{4^x - 3^x}{5^x - 1}$ ans: $\log \left(\frac{4}{3}\right)$
 $\log 5$

03. $\lim_{x \rightarrow 0} \frac{6^x - 3^x}{4^x - 1}$ ans : $1/2$

$$04. \lim_{x \rightarrow 0} \frac{5^x - 3^x}{4^x - 2^x} \quad \text{ans: } \log \left[\frac{5}{3} \right]$$

$$\frac{\log 2}{\log 2}$$

$$04. \lim_{x \rightarrow 0} \frac{15^x - 3^x - 5^x + 1}{x \cdot \tan x} \quad \text{ans : } \log 3 \cdot \log 5$$

$$05. \lim_{x \rightarrow 0} \frac{a^{2x} - b^x}{x} \quad \text{ans: } \log \left[\frac{a^2}{b} \right]$$

$$05. \lim_{x \rightarrow 0} \frac{12^x - 3^x - 4^x + 1}{1 - \cos 2x} \quad \text{ans : } \log 3 \cdot \log 2$$

$$06. \lim_{x \rightarrow 0} \frac{2^x + 3^x + 4^x - 3}{x} \quad \text{ans : } \log 24$$

Q SET - 5

$$01. \lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2} \quad \text{ans : } (\log a)^2$$

$$07. \lim_{x \rightarrow 0} \frac{2^x + 5^x + 7^x - 3}{x} \quad \text{ans : } \log 70$$

$$02. \lim_{x \rightarrow 0} \frac{5^x + 5^{-x} - 2}{x^2} \quad \text{ans : } (\log 5)^2$$

$$08. \lim_{x \rightarrow 0} \frac{5^x + 3^x - 2^x - 1}{x} \quad \text{ans: } \log \left[\frac{15}{2} \right]$$

$$03. \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{\cos 3x - \cos 5x} \quad \text{ans : } 1/8$$

$$09. \lim_{x \rightarrow 0} \frac{4^x + 5^x - 2^{x+1}}{x} \quad \text{ans : } \log 5$$

$$04. \lim_{x \rightarrow 0} \frac{5^x + 5^{-x} - 2}{\cos 2x - \cos 6x} \quad \text{ans : } \frac{(\log 5)^2}{16}$$

$$10. \lim_{x \rightarrow 0} \frac{a^x + b^x - 2^{x+1}}{x} \quad \text{ans: } \log \left[\frac{ab}{4} \right]$$

Q SET - 6

$$01. \lim_{x \rightarrow 3} \frac{1}{(x-3)} \quad \text{ans : } e$$

$$11. \lim_{x \rightarrow 0} \frac{2^x + 3^x + 4^x - 3^{x+1}}{x} \quad \text{ans: } \log \left[\frac{8}{9} \right]$$

$$02. \lim_{x \rightarrow 4} \frac{1}{(x-4)} \quad \text{ans : } e$$

Q SET - 4

$$01. \lim_{x \rightarrow 0} \frac{10^x - 5^x - 2^x + 1}{x^2} \quad \text{ans : } \log 2 \cdot \log 5$$

$$03. \lim_{x \rightarrow 3} \frac{\log x - \log 3}{x - 3} \quad \text{ans : } 1/3$$

$$02. \lim_{x \rightarrow 0} \frac{15^x - 3^x - 5^x + 1}{x^2} \quad \text{ans : } \log 3 \cdot \log 5$$

$$04. \lim_{x \rightarrow 3} \frac{\log x - \log 3}{x^2 - 9} \quad \text{ans : } 1/18$$

$$03. \lim_{x \rightarrow 0} \frac{21^x - 3^x - 7^x + 1}{x^2} \quad \text{ans : } \log 3 \cdot \log 7$$

SOLUTION TO Q SET - 1

$$01. \lim_{x \rightarrow 0} \left(1 + \frac{x}{3}\right)^{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0} \left\{ \left(1 + \frac{x}{3}\right)^{\frac{3}{x}} \right\}^{\frac{1}{3}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{\frac{1}{3}}$$

$$02. \lim_{x \rightarrow 0} \left(1 + \frac{x}{4}\right)^{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0} \left\{ \left(1 + \frac{x}{4}\right)^{\frac{4}{x}} \right\}^{\frac{1}{4}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{\frac{1}{4}}$$

$$03. \lim_{x \rightarrow 0} \left(1 + \frac{2x}{3}\right)^{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0} \left\{ \left(1 + \frac{2x}{3}\right)^{\frac{3}{2x}} \right\}^{\frac{2}{3}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{\frac{2}{3}}$$

$$04. \lim_{x \rightarrow 0} \left(1 + \frac{3x}{4}\right)^{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0} \left\{ \left(1 + \frac{3x}{4}\right)^{\frac{4}{3x}} \right\}^{\frac{3}{4}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{\frac{3}{4}}$$

$$05. \lim_{x \rightarrow 0} (1 + 2x)^{\frac{3}{x}}$$

$$= \lim_{x \rightarrow 0} \left((1 + 2x)^{\frac{1}{2x}} \right)^{2.3}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^6$$

$$06. \lim_{x \rightarrow 0} (1 + 2x)^{\frac{5}{x}}$$

$$= \lim_{x \rightarrow 0} \left((1 + 2x)^{\frac{1}{2x}} \right)^{2.5}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{10}$$

$$07. \lim_{x \rightarrow 0} \left(\frac{1 + 4x}{1 - 4x} \right)^{\frac{1}{x}}$$

SPLIT THE POWER

$$= \lim_{x \rightarrow 0} \frac{(1 + 4x)^{\frac{1}{x}}}{(1 - 4x)^{\frac{1}{x}}}$$

$$= \lim_{x \rightarrow 0} \frac{\left((1 + 4x)^{\frac{1}{4x}} \right)^4}{\left((1 - 4x)^{\frac{-1}{4x}} \right)^{-4}}$$

SET THE POWER

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= \frac{e^4}{e^{-4}}$$

$$= e^{4+4} = e^8$$

$$09. \lim_{x \rightarrow 0} \left(\frac{1 + 8x}{1 - 8x} \right)^{\frac{1}{x}}$$

SPLIT THE POWER

$$= \lim_{x \rightarrow 0} \frac{(1 + 8x)^{\frac{1}{x}}}{(1 - 8x)^{\frac{1}{x}}}$$

$$= \lim_{x \rightarrow 0} \frac{\left((1 + 8x)^{\frac{1}{8x}} \right)^8}{\left((1 - 8x)^{\frac{-1}{8x}} \right)^{-8}}$$

SET THE POWER

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= \frac{e^8}{e^{-8}}$$

$$= e^{8+8} = e^{16}$$

$$08. \lim_{x \rightarrow 0} \left(\frac{1 - 5x}{1 + 5x} \right)^{\frac{1}{x}}$$

SPLIT THE POWER

$$= \lim_{x \rightarrow 0} \frac{(1 - 5x)^{\frac{1}{x}}}{(1 + 5x)^{\frac{1}{x}}}$$

$$= \lim_{x \rightarrow 0} \frac{\left((1 - 5x)^{\frac{-1}{5x}} \right)^{-5}}{\left((1 + 5x)^{\frac{1}{5x}} \right)^5}$$

SET THE POWER

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= \frac{e^{-5}}{e^5}$$

$$= e^{-5-5} = e^{-10}$$

$$10. \lim_{x \rightarrow 0} \left(\frac{1 - 3x}{1 + 4x} \right)^{\frac{1}{x}}$$

SPLIT THE POWER

$$= \lim_{x \rightarrow 0} \frac{(1 - 3x)^{\frac{1}{x}}}{(1 + 4x)^{\frac{1}{x}}}$$

$$= \lim_{x \rightarrow 0} \frac{\left((1 - 3x)^{\frac{-1}{3x}} \right)^{-3}}{\left((1 + 4x)^{\frac{1}{4x}} \right)^4}$$

SET THE POWER

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= \frac{e^{-3}}{e^4}$$

$$= e^{-3-4} = e^{-7}$$

SOLUTION TO Q SET - 2

$$11. \lim_{x \rightarrow 0} \left(\frac{4+x}{4-x} \right)^{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\frac{4+x}{4}}{\frac{4-x}{4}} \right)^{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0} \left(\frac{1 + \frac{x}{4}}{1 - \frac{x}{4}} \right)^{\frac{1}{x}}$$

SPLIT THE POWER

$$= \lim_{x \rightarrow 0} \frac{\left(1 + \frac{x}{4} \right)^{\frac{1}{x}}}{\left(1 - \frac{x}{4} \right)^{\frac{1}{x}}}$$

SET THE POWER

$$= \lim_{x \rightarrow 0} \frac{\left\{ \left(1 + \frac{x}{4} \right)^{\frac{4}{x}} \right\}^{\frac{1}{4}}}{\left\{ \left(1 - \frac{x}{4} \right)^{\frac{-4}{x}} \right\}^{\frac{-1}{4}}}$$

$$= \frac{e^{\frac{1}{4}}}{e^{\frac{-1}{4}}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{\frac{1}{4} + \frac{1}{4}}$$

$$= e^{\frac{1}{2}}$$

$$01. \lim_{x \rightarrow 0} \frac{\log(1+2x)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2 \log(1+2x)}{2x}$$

LIMIT JATHI FORMULA AATI

$$= 2(1)$$

$$= 2$$

$$02. \lim_{x \rightarrow 0} \frac{\log(1+6x)}{2x}$$

$$= \lim_{x \rightarrow 0} \frac{1}{2} \frac{\log(1+6x)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{6}{2} \frac{\log(1+6x)}{6x}$$

LIMIT JATHI FORMULA AATI

$$= 3(1)$$

$$= 3$$

$$03. \lim_{x \rightarrow 0} \frac{1}{x} \log \left(1 + \frac{8x}{3} \right)$$

$$= \lim_{x \rightarrow 0} \frac{\log \left(1 + \frac{8x}{3} \right)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{8}{3} \log \left(1 + \frac{8x}{3} \right)}{\frac{8x}{3}}$$

LIMIT JATHI FORMULA AATI

$$= \frac{8}{3} (1)$$

$$= \frac{8}{3}$$

SOLUTION TO Q SET - 3

$$04. \quad \lim_{x \rightarrow 0} \frac{\log(1+5x) - \log(1+3x)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\log(1+5x)}{x} - \frac{\log(1+3x)}{x}$$

$$= \lim_{x \rightarrow 0} 5 \frac{\log(1+5x)}{5x} - 3 \frac{\log(1+3x)}{3x}$$

LIMIT JATHI FORMULA AATI

$$= 5(1) - 3(1) = 2$$

$$05. \quad \lim_{x \rightarrow 0} \frac{\log 7 + \log\left(\frac{x+1}{7}\right)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\log\left(7 \cdot \frac{x+1}{7}\right)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\log(1+x)}{x}$$

LIMIT JATHI FORMULA AATI

$$= 1$$

$$06. \quad \lim_{x \rightarrow 0} \frac{\log(2+x) - \log 2}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\log\left(\frac{2+x}{2}\right)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\log\left(1 + \frac{x}{2}\right)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{2} \log\left(1 + \frac{x}{2}\right)}{\frac{x}{2}}$$

LIMIT JATHI FORMULA AATI

$$= \frac{1}{2} (1) = \frac{1}{2}$$

$$01. \quad \lim_{x \rightarrow 0} \frac{5^x - 4^x}{x}$$

$$= \lim_{x \rightarrow 0} \frac{5^x - 1 - 4^x + 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{5^x - 1 - (4^x - 1)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{5^x - 1}{x} - \frac{4^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 5 - \log 4$$

$$= \log\left(\frac{5}{4}\right)$$

$$02. \quad \lim_{x \rightarrow 0} \frac{4^x - 3^x}{5^x - 1}$$

$$= \lim_{x \rightarrow 0} \frac{4^x - 1 - 3^x + 1}{5^x - 1}$$

$$= \lim_{x \rightarrow 0} \frac{4^x - 1 - (3^x - 1)}{5^x - 1}$$

Dividing Numerator & Denominator by x ,
as $x \rightarrow 0$, $x \neq 0$

$$= \lim_{x \rightarrow 0} \frac{\frac{4^x - 1 - (3^x - 1)}{x}}{\frac{5^x - 1}{x}}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{4^x - 1}{x} - \frac{3^x - 1}{x}}{\frac{5^x - 1}{x}}$$

LIMIT JATHI FORMULA AATI

$$= \frac{\log 4 - \log 3}{\log 5}$$

$$= \frac{\log\left(\frac{4}{3}\right)}{\log 5}$$

$$03. \quad \lim_{x \rightarrow 0} \frac{6^x - 3^x}{4^x - 1}$$

$$= \lim_{x \rightarrow 0} \frac{6^x - 1 - 3^x + 1}{4^x - 1}$$

$$= \lim_{x \rightarrow 0} \frac{6^x - 1 - (3^x - 1)}{4^x - 1}$$

Dividing Numerator & Denominator by x ,
as $x \rightarrow 0$, $x \neq 0$

$$= \lim_{x \rightarrow 0} \frac{\frac{6^x - 1 - (3^x - 1)}{x}}{\frac{4^x - 1}{x}}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{6^x - 1}{x} - \frac{3^x - 1}{x}}{\frac{4^x - 1}{x}}$$

LIMIT JATHI FORMULA AATI

$$= \frac{\log 6 - \log 3}{\log 4}$$

$$= \frac{\log\left(\frac{6}{3}\right)}{\log 4}$$

$$= \frac{\log 2}{\log 4}$$

$$= \frac{\log 2}{\log 2^2}$$

$$= \frac{\log 2}{2\log 2}$$

$$= \frac{1}{2}$$

$$04. \quad \lim_{x \rightarrow 0} \frac{5^x - 3^x}{4^x - 2^x}$$

$$= \lim_{x \rightarrow 0} \frac{5^x - 1 - 3^x + 1}{4^x - 1 - 2^x + 1}$$

$$= \lim_{x \rightarrow 0} \frac{5^x - 1 - (3^x - 1)}{4^x - 1 - (2^x - 1)}$$

Dividing Numerator & Denominator by x ,
as $x \rightarrow 0$, $x \neq 0$

$$= \lim_{x \rightarrow 0} \frac{\frac{5^x - 1 - (3^x - 1)}{x}}{\frac{4^x - 1 - (2^x - 1)}{x}}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{5^x - 1}{x} - \frac{3^x - 1}{x}}{\frac{4^x - 1}{x} - \frac{2^x - 1}{x}}$$

LIMIT JATHI FORMULA AATI

$$= \frac{\log 5 - \log 3}{\log 4 - \log 2}$$

$$= \frac{\log\left(\frac{5}{3}\right)}{\log\left(\frac{4}{2}\right)}$$

$$= \frac{\log\left(\frac{5}{3}\right)}{\log 2}$$

$$05. \quad \lim_{x \rightarrow 0} \frac{a^{2x} - b^x}{x}$$

$$= \lim_{x \rightarrow 0} \frac{a^{2x} - 1 - b^x + 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{a^{2x} - 1 - (b^x - 1)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{a^{2x} - 1}{x} - \frac{b^x - 1}{x}$$

$$= \lim_{x \rightarrow 0} 2 \frac{a^{2x} - 1}{2x} - \frac{b^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= 2 \log a - \log b$$

$$= \log a^2 - \log b$$

$$= \log \left(\frac{a^2}{b} \right)$$

$$06. \lim_{x \rightarrow 0} \frac{2^x + 3^x + 4^x - 3}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2^x - 1 + 3^x - 1 + 4^x - 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2^x - 1}{x} + \frac{3^x - 1}{x} + \frac{4^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 2 + \log 3 + \log 4$$

$$= \log (2 \times 3 \times 4)$$

$$= \log 24$$

$$07. \lim_{x \rightarrow 0} \frac{2^x + 5^x + 7^x - 3}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2^x - 1 + 5^x - 1 + 7^x - 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2^x - 1}{x} + \frac{5^x - 1}{x} + \frac{7^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 2 + \log 5 + \log 7$$

$$= \log (2 \times 5 \times 7)$$

$$= \log 70$$

$$08. \lim_{x \rightarrow 0} \frac{5^x + 3^x - 2^x - 1}{x}$$

EK KA BHALA SUM SE KAR, DO KA BHALA KHUD SE KAR

$$= \lim_{x \rightarrow 0} \frac{5^x - 1 + 3^x - 2^x}{x}$$

$$= \lim_{x \rightarrow 0} \frac{5^x - 1 + 3^x - 1 - 2^x + 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{5^x - 1 + 3^x - 1 - (2^x - 1)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{5^x - 1}{x} + \frac{3^x - 1}{x} - \frac{2^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 5 + \log 3 - \log 2$$

$$= \log \left(\frac{5 \times 3}{2} \right)$$

$$= \log \left(\frac{15}{2} \right)$$

$$09. \lim_{x \rightarrow 0} \frac{4^x + 5^x - 2^{x+1}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{4^x + 5^x - 2^x \cdot 2}{x}$$

$$= \lim_{x \rightarrow 0} \frac{4^x - 1 + 5^x - 1 - 2^x \cdot 2 + 2}{x}$$

$$= \lim_{x \rightarrow 0} \frac{4^x - 1 + 5^x - 1 - 2(2^x - 1)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{4^x - 1}{x} + \frac{5^x - 1}{x} - \frac{2(2^x - 1)}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 4 + \log 5 - 2\log 2$$

$$= \log 4 + \log 5 - \log 4$$

$$= \log 5$$

$$10. \lim_{x \rightarrow 0} \frac{a^x + b^x - 2^{x+1}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{a^x + b^x - 2^x \cdot 2}{x}$$

$$= \lim_{x \rightarrow 0} \frac{a^x - 1 + b^x - 1 - 2^x \cdot 2 + 2}{x}$$

$$= \lim_{x \rightarrow 0} \frac{a^x - 1 + b^x - 1 - 2(2^x - 1)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{a^x - 1}{x} + \frac{b^x - 1}{x} - \frac{2(2^x - 1)}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log a + \log b - 2\log 2$$

$$= \log a + \log b - \log 4$$

$$= \log \left(\frac{ab}{4} \right)$$

$$11. \lim_{x \rightarrow 0} \frac{2^x + 3^x + 4^x - 3^{x+1}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2^x + 3^x + 4^x - 3^x \cdot 3}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2^x - 1 + 3^x - 1 + 4^x - 1 - 3^x \cdot 3 + 3}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2^x - 1 + 3^x - 1 + 4^x - 1 - 3(3^x - 1)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2^x - 1}{x} + \frac{3^x - 1}{x} + \frac{4^x - 1}{x} - \frac{3(3^x - 1)}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 2 + \log 3 + \log 4 - 3\log 3$$

$$= \log 2 + \log 3 + \log 4 - \log 27$$

$$= \log \left(\frac{2 \times 3 \times 4}{27} \right)$$

$$= \log \left(\frac{8}{9} \right)$$

SOLUTION TO Q SET - 4

$$01. \lim_{x \rightarrow 0} \frac{10^x - 5^x - 2^x + 1}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{5^x \cdot 2^x - 5^x - 2^x + 1}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{5^x (2^x - 1) - 1(2^x - 1)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(5^x - 1)(2^x - 1)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{5^x - 1}{x} \cdot \frac{2^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 5 \cdot \log 2$$

$$02. \lim_{x \rightarrow 0} \frac{15^x - 3^x - 5^x + 1}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{5^x \cdot 3^x - 3^x - 5^x + 1}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{3^x (5^x - 1) - 1(5^x - 1)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(3^x - 1)(5^x - 1)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{3^x - 1}{x} \cdot \frac{5^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 3 \cdot \log 5$$

$$03. \lim_{x \rightarrow 0} \frac{21^x - 3^x - 7^x + 1}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{7^x 3^x - 3^x - 7^x + 1}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{3^x (7^x - 1) - 1(7^x - 1)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(3^x - 1)(7^x - 1)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{3^x - 1}{x} \cdot \frac{7^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 3 \cdot \log 7$$

$$04. \lim_{x \rightarrow 0} \frac{15^x - 3^x - 5^x + 1}{x \cdot \tan x}$$

$$= \lim_{x \rightarrow 0} \frac{5^x 3^x - 3^x - 5^x + 1}{x \cdot \tan x}$$

$$= \lim_{x \rightarrow 0} \frac{3^x (5^x - 1) - 1(5^x - 1)}{x \cdot \tan x}$$

$$= \lim_{x \rightarrow 0} \frac{(3^x - 1)(5^x - 1)}{x \cdot \tan x}$$

Dividing Numerator & Denominator by x^2 , $x \rightarrow 0$, $x \neq 0$

$$= \lim_{x \rightarrow 0} \frac{\frac{(3^x - 1)(5^x - 1)}{x^2}}{\frac{x \cdot \tan x}{x^2}}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{3^x - 1}{x} \cdot \frac{5^x - 1}{x}}{\frac{\tan x}{x}}$$

LIMIT JATHI FORMULA AATI

$$= \frac{\log 3 \cdot \log 5}{(1)}$$

$$= \log 3 \cdot \log 5$$

$$05. \lim_{x \rightarrow 0} \frac{12^x - 3^x - 4^x + 1}{1 - \cos 2x}$$

$$= \lim_{x \rightarrow 0} \frac{4^x 3^x - 3^x - 4^x + 1}{2 \sin^2 x}$$

$$= \lim_{x \rightarrow 0} \frac{3^x (4^x - 1) - 1(4^x - 1)}{2 \sin^2 x}$$

$$= \lim_{x \rightarrow 0} \frac{(3^x - 1)(4^x - 1)}{2 \sin^2 x}$$

Dividing Numerator & Denominator by x^2 , $x \rightarrow 0$, $x \neq 0$

$$= \lim_{x \rightarrow 0} \frac{\frac{(3^x - 1)(4^x - 1)}{x^2}}{\frac{2 \sin^2 x}{x^2}}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{3^x - 1}{x} \cdot \frac{4^x - 1}{x}}{2 \left(\frac{\sin x}{x} \right)^2}$$

DISTRIBUTE
SQUARE SQUARE
THE WHOLE SQUARE

LIMIT JATHI FORMULA AATI

$$= \frac{\log 3 \cdot \log 4}{2(1)^2}$$

$$= \frac{\log 3 \cdot 2 \log 2}{2}$$

$$= \log 3 \cdot \log 2$$

SOLUTION TO Q SET - 5

$$01. \quad \lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{a^x + \frac{1}{a^x} - 2}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(a^x)^2 + 1 - 2 \cdot a^x}{a^x \cdot x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(a^x - 1)^2}{x^2} \cdot \frac{1}{a^x}$$

$$= \lim_{x \rightarrow 0} \left(\frac{a^x - 1}{x} \right)^2 \cdot \frac{1}{a^x} \quad \begin{array}{l} \text{SQUARE SQUARE} \\ \text{THE WHOLE SQUARE} \end{array}$$

LIMIT JATHI , KAHI FORMULA LAGTI , KAHI PASTE HOTI

$$= (\log a)^2 \cdot \frac{1}{a^0}$$

$$= (\log a)^2$$

$$02. \quad \lim_{x \rightarrow 0} \frac{5^x + 5^{-x} - 2}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{5^x + \frac{1}{5^x} - 2}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(5^x)^2 + 1 - 2 \cdot 5^x}{5^x \cdot x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(5^x - 1)^2}{x^2} \cdot \frac{1}{5^x}$$

$$= \lim_{x \rightarrow 0} \left(\frac{5^x - 1}{x} \right)^2 \cdot \frac{1}{5^x} \quad \begin{array}{l} \text{SQUARE SQUARE} \\ \text{THE WHOLE SQUARE} \end{array}$$

LIMIT JATHI , KAHI FORMULA LAGTI , KAHI PASTE HOTI

$$= (\log 5)^2 \cdot \frac{1}{5^0}$$

$$= (\log 5)^2$$

$$03. \quad \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{\cos 3x - \cos 5x}$$

$$= \lim_{x \rightarrow 0} \frac{e^x + \frac{1}{e^x} - 2}{-2 \sin \left[\frac{3x+5x}{2} \right] \cdot \sin \left[\frac{3x-5x}{2} \right]}$$

$$= \lim_{x \rightarrow 0} \frac{(e^x)^2 + 1 - 2 \cdot e^x}{e^x \cdot (-2 \sin 4x \cdot \sin(-x))}$$

$$= \lim_{x \rightarrow 0} \frac{(e^x - 1)^2}{e^x \cdot 2 \sin 4x \cdot \sin x}$$

Dividing Numerator & Denominator by x^2 , $x \rightarrow 0$, $x \neq 0$

$$= \lim_{x \rightarrow 0} \frac{\frac{(e^x - 1)^2}{x^2} \cdot \frac{1}{e^x}}{2 \frac{\sin 4x}{x} \cdot \frac{\sin x}{x}}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{(e^x - 1)^2}{x^2} \cdot \frac{1}{e^x}}{2 \frac{\sin 4x}{x} \cdot \frac{\sin x}{x}} \quad \text{DISTRIBUTE}$$

$$= \lim_{x \rightarrow 0} \frac{\left(\frac{e^x - 1}{x} \right)^2 \cdot \frac{1}{e^x}}{2.4 \frac{\sin 4x}{4x} \cdot \frac{\sin x}{x}} \quad \begin{array}{l} \text{SQUARE SQUARE} \\ \text{THE WHOLE SQUARE} \end{array}$$

LIMIT JATHI , KAHI FORMULA LAGTI , KAHI PASTE HOTI

$$= \frac{(\log e)^2 \cdot \frac{1}{e^0}}{2.4 \cdot (1) \cdot (1)} = \frac{1}{8}$$

SOLUTION TO Q SET - 6

$$\begin{aligned}
 04. \quad \lim_{x \rightarrow 0} \frac{5^x + 5^{-x} - 2}{\cos 2x - \cos 6x} \\
 &= \lim_{x \rightarrow 0} \frac{5^x + \frac{1}{5^x} - 2}{-2 \sin\left(\frac{2x+6x}{2}\right) \cdot \sin\left(\frac{2x-6x}{2}\right)} \\
 &= \lim_{x \rightarrow 0} \frac{(5^x)^2 + 1 - 2 \cdot 5^x}{5^x} \\
 &\quad - 2 \sin 4x \cdot \sin(-2x) \\
 &= \lim_{x \rightarrow 0} \frac{(5^x - 1)^2}{5^x} \\
 &\quad 2 \sin 4x \cdot \sin 2x
 \end{aligned}$$

Dividing Numerator & Denominator by x^2 , $x \rightarrow 0$, $x \neq 0$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{\frac{(5^x - 1)^2}{5^x \cdot x^2}}{\frac{2 \sin 4x \cdot \sin 2x}{x^2}} \\
 &= \lim_{x \rightarrow 0} \frac{\frac{(5^x - 1)^2}{x^2} \cdot \frac{1}{5^x}}{2 \frac{\sin 4x}{x} \cdot \frac{\sin 2x}{x}} \quad \text{DISTRIBUTE} \\
 &= \lim_{x \rightarrow 0} \frac{\left[\frac{5^x - 1}{x}\right]^2 \cdot \frac{1}{5^x}}{2.4 \frac{\sin 4x}{4x} \cdot 2 \frac{\sin 2x}{2x}} \quad \begin{array}{l} \text{SQUARE SQUARE} \\ \text{THE WHOLE SQUARE} \end{array}
 \end{aligned}$$

LIMIT JATHI , KAHI FORMULA LAGTI , KAHI PASTE HOTI

$$= \frac{(\log 5)^2 \cdot \frac{1}{5^0}}{2.4 \cdot (1) \cdot 2(1)}$$

$$= \frac{(\log 5)^2}{16}$$

$$\begin{aligned}
 01. \quad \lim_{x \rightarrow 3} \frac{1}{(x-3)} \\
 \lim_{x \rightarrow 3} (x-2) \\
 x = 3 + h \\
 \lim_{h \rightarrow 0} \frac{1}{3+h-3} \\
 = \lim_{h \rightarrow 0} (3+h-2) \\
 \lim_{h \rightarrow 0} \frac{1}{h} \\
 = \lim_{h \rightarrow 0} (1+h) \\
 = e
 \end{aligned}$$

$$\begin{aligned}
 02. \quad \lim_{x \rightarrow 4} \frac{1}{(x-4)} \\
 \lim_{x \rightarrow 4} (x-3) \\
 x = 4 + h \\
 \lim_{h \rightarrow 0} \frac{1}{4+h-4} \\
 = \lim_{h \rightarrow 0} (4+h-3) \\
 \lim_{h \rightarrow 0} \frac{1}{h} = e \\
 = \lim_{h \rightarrow 0} (1+h)
 \end{aligned}$$

$$\begin{aligned}
 03. \quad \lim_{x \rightarrow 3} \frac{\log x - \log 3}{x-3} \\
 x = 3 + h \\
 = \lim_{h \rightarrow 0} \frac{\log(3+h) - \log 3}{3+h-3}
 \end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{\log\left[\frac{3+h}{3}\right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\log\left[1 + \frac{h}{3}\right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{3} \log \left(1 + \frac{h}{3} \right)}{\frac{h}{3}}$$

LIMIT JATHI FORMULA AATI

$$= \frac{1}{3} (1)$$

$$= \frac{1}{3}$$

$$04. \lim_{x \rightarrow 3} \frac{\log x - \log 3}{x^2 - 9}$$

$$= \lim_{x \rightarrow 3} \frac{\log x - \log 3}{x - 3} \cdot \frac{1}{x + 3}$$

$$x = 3 + h$$

$$= \lim_{h \rightarrow 0} \frac{\log (3 + h) - \log 3}{3 + h - 3} \cdot \frac{1}{6 + h}$$

$$= \lim_{h \rightarrow 0} \frac{\log \left(\frac{3 + h}{3} \right)}{h} \cdot \frac{1}{6 + h}$$

$$= \lim_{h \rightarrow 0} \frac{\log \left(1 + \frac{h}{3} \right)}{h} \cdot \frac{1}{6 + h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{3} \log \left(1 + \frac{h}{3} \right)}{\frac{h}{3}} \cdot \frac{1}{6 + h}$$

LIMIT JATHI , KAHİ FORMULA LAGTI , KAHİ PASTE HOTI

$$= \frac{1}{3} (1) \cdot \frac{1}{6 + 0}$$

$$= \frac{1}{18}$$